# Inequality and Asset Prices SED 2012

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# Question

# How does wealth and income inequality affect asset prices?



## Recent Developments

- "The 1%": A small group of investors holds ever larger share of asset market.
- Some groups have ceased to participate in asset markets, e.g. have become liquidity-constrained.
- Income inequality has increased.
- ⇒ What does this mean for asset prices?
- $\Rightarrow$  Interesting: Debate about re-distributive tax policy, which will affect inequality.



### Potential Channels

How could changes in inequality affect asset prices?

#### In this paper:

- The rich's income is highly correlated with asset returns
  - $\Rightarrow$  Value payoffs from assets different from representative agent
- Changes in market participation: poor are driven out of market
  - $\Rightarrow$  Changes in aggregate asset demand and supply

### Not in this paper:

- The rich have different information
- Preferences:
  - Rich are more patient...: e.g. Krusell & Smith (1997)
  - ... or less risk averse: e.g. Dumas (1989)
  - Concave absolute risk tolerance: e.g. Gollier (2001)
  - Keeping-up-with-the-Joneses preferences: e.g. Johnson (2011)



Literature Data Model Results Conclusions Extra Slides

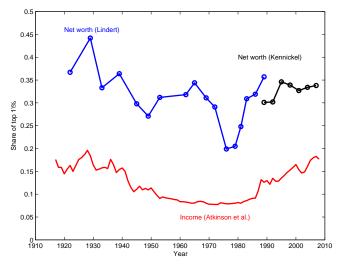
#### Literature

- Bewley-Krusell-Smith-type models:
  - Idiosyncratic shocks wash out.
  - Wealth distribution does not affect asset prices.
  - ⇒ Need correlated risk to generate effects.
- Incomplete-markets with few types of agents: DenHaan (1996), Heaton & Lucas (1996), Stepanchuk & Tsyrennikov (2011) etc.
  - $\Rightarrow$  Highly computational, focus on long-run time-series outcomes.
- 3 Closest to our model: Scheinkman & Weiss (1986), but:
  - Agents never cease to participate in asset market (Inada condition).
  - Linear preferences for leisure ⇒ Rich agents have constant consumption.
  - ⇒ We obtain different results for asset prices in a more standard environment.



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# Data: Inequality





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# Issues with Inequality Data

- Good data only since 1970s (especially for "99%")
- Wealth-inequality series poorly measured.
- Slow-moving ⇒ Essentially very few observations.
- Other factors that affect asset markets co-move with inequality: business cycle, policy,...
  - $\Rightarrow$  Hard to identify contribution of inequality.
- Exploit cross-country variation? Not convincing: asset markets correlate.
- ⇒ We opt for modeling approach. ◆ asset data



Model

**Endowment economy** with two types of agents 1, 2 (of equal measure) in continuous time:

- Exogenous endowment stream:
  - Agent 1's income  $y \in \{y_l, y_h\}$  follows Poisson process with switching rate  $\eta$ .
  - Whenever agent 1 has low income  $y_l$ , agent 2 has high income  $y_h$  and vice versa.
- One asset: Lucas tree in unit supply
  - Initial endowments:  $a_0^1, a_0^2$ .
  - **Constant dividend stream** d > 0.
  - No short-selling:  $a_t^i \ge \underline{a} = 0$ .
  - Traded at price  $P_t$  (in terms of consumption good).
  - $\Rightarrow$  GDP is constant (normalize:  $d + y_l + y_h = 1$ ).
- Standard preferences:

$$U_i = E_0 \int_0^\infty e^{-\rho t} u(c_t^i) dt.$$



# Incomplete Markets: Agent's Constraints

■ The budget constraint at t, given the inherited asset position  $a_{t-\Delta t}$ , is:

$$c_t \Delta t + P_t \left( \underbrace{a_t - a_{t-\Delta t}}_{\text{net demand for tree at } t} \right) = \underbrace{y_t \Delta t + a_{t-\Delta t} d\Delta t}_{\text{total income}}.$$

■ Divide by  $\Delta t$  and take limits as  $\Delta t \rightarrow 0$  to find budget constraint:

$$c_t + P_t \dot{a}_t = y_t + da_t,$$

where  $\dot{a}_t = \lim_{\Delta t \to 0} \frac{a_t - a_{t-\Delta t}}{\Delta t}$  is the agent's **net demand** for the asset at t.

No short-selling:

$$a_t \geq 0$$
.



- Aggregate state:  $(y_t, A_t)$
- Individual state:  $(y_t, A_t; a_t)$
- $A_t$ : Fraction of tree that **all** agents of type 1 hold (together).
  - $\Rightarrow$  Must have  $A_t \in [0,1]$ .
- **Perceived law of motion**  $A_t$  implied by:
  - 1  $C(y_t, A_t)$ : Consumption of typical agent 1 and
  - $P(y_t, A_t)$ : Pricing function.
- The typical agent 2 consumes  $1 C(y_t, A_t)$ .



A competitive equilibrium consists of functions  $c^1(y, A)$ ,  $c^2(y, A)$ , C(y, A) and P(y, A) such that

- The stochastic process  $c^i(y_t, A_t)$  solves agent i's problem given the perceived law of motion implied by C(y, A) and P(y, A).
- Rational expectations/consistency:

$$c^{1}(y, A) = C(y, A),$$
  
 $c^{2}(y, A) = 1 - C(y, A).$ 

Market clearing for consumption good (already implied):

$$c^{1}(y,A) + c^{2}(y,A) = d + y_{I} + y_{h} = 1.$$

**Note:** Asset-market clearing is implied by Walras' Law.



**Strategy:** Find functions C(y, A) and P(y, A) such that

- **1** Stochastic process  $c_t^1 = C(y_t, A_t)$  fulfills agent 1's Euler equation.
- 2 Stochastic process  $c_t^2 = 1 C(y_t, A_t)$  fulfills agent 2's Euler equation.

 $\Rightarrow$  4 first-order differential equations for

$$\{C(\cdot, y_l), C(\cdot, y_h), P(\cdot, y_l), P(\cdot, y_h)\}$$

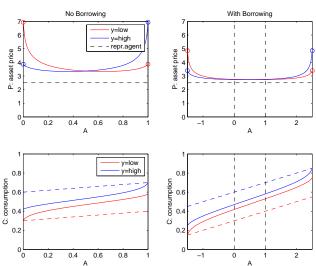
with 4 boundary conditions (Euler equations at constraints).



## Results

## Results: numerical example

$$\gamma=$$
 2 (CRRA),  $\rho=$  0.04,  $\eta=$  0.1,  $y_{l}=$  0.3,  $y_{h}=$  0.6 and  $d=$  0.1.





# Results (I)

- Asset prices are highest when constraint  $A \in \{0,1\}$  is reached. Poor agent does not participate in asset market: drop in supply.  $\Rightarrow$  Rich agent prices asset, wants insurance: surge in demand. Fears:
  - 1 Drop in labor income.
  - 2 Drop in asset price and thus wealth.
- **Expected returns** are **lowest** when **constraint** is reached.
  - Only rich agent demands asset, wants insurance.
- Asset prices are increasing in wealth inequality.
   High prices at constraint feed back into interior.
   ⇒ Prices high whenever reaching constraint appears likely.



# Results (II)

- Fixing wealth inequality (A), asset prices are high when asset-rich agent is also income-rich, i.e.
  - when income inequality is high, and
  - when wealth inequality is on the rise.
  - Intuition: higher probability of hitting constraint
- Asset-price volatility is highest when wealth inequality is large.
  - $\Rightarrow$  Intuition: Large movements in asset demand since participation changes!

# **Pricing Equation**

- Average marginal utility:  $\Gamma_t \equiv \frac{1}{2}u'(c_t) + \frac{1}{2}u'(1-c_t)$  $\Rightarrow$  Strictly increasing in consumption inequality if  $u'(\cdot)$  convex
- Stochastic discount factor:

$$\Lambda_t = \Gamma_t \underbrace{\xi^{I_t^1}}_{>1} \underbrace{\zeta^{I_t^2}}_{>1},$$

 $l_t^j$ : the number of times an income reversal occurred when agent i was constrained up.

■ The price of the asset satisfies

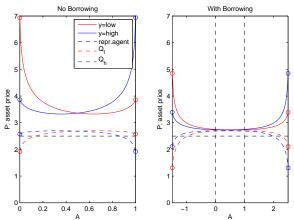
$$P_t \Lambda_t = E_t \left[ \int_t^\infty \left( e^{-\rho(s-t)} \Lambda_{t+s} d \right) ds \right].$$

- 1 Constraint likely to bind in near future:  $P_t \uparrow$
- 2 Relatively high current consumption inequality:  $P_t \downarrow$



# Disentangling the Pricing Equation

Construct hypothetical price:  $Q_t\Gamma_t = E_t \int_0^\infty (e^{-\rho s}\Gamma_s d) ds$ 



⇒ High current consumption inequality lowers asset price, but effect from constraint dominates 4 □ > 4 □ > 4 □ > 4 □ >



## Conclusions

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## Conclusions

- Main results: When inequality is high,
  - 1 asset prices are high.
  - 2 asset returns are low.
  - volatility is high.
- Find similar results when replacing Lucas tree by:
  - 1 short-term bond
  - 2 capital in production economy
- Continuous time allows interesting characterizations at constraint.
- To do: policy. How does income redistribution affect asset markets?



## Further lessons

- Stark counterexample to Krusell & Smith (1998):
   Wealth distribution matters a lot for predicting prices here.
- Cautionary note for computation in incomplete-markets international-finance models:
   Prices can do weird things when constraints start to bind.



#### Extra Slides

# Data summary: inequality and asset returns

Returns data: Risk-free rate and S&P 500, 1917-2010.

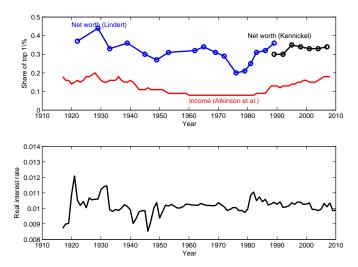
- No visible correlation of inequality and asset-returns.
- Asset returns seem more volatile in times of high inequality.

**But:** Don't want to take this evidence too seriously because of issues mentioned above.



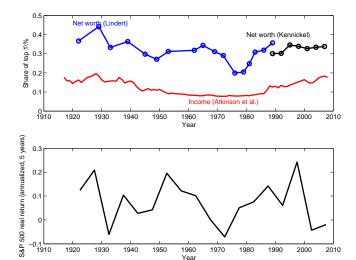
tivation Literature Data Model Results Conclusions **Extra Slides** 

# Inequality and the risk-free rate



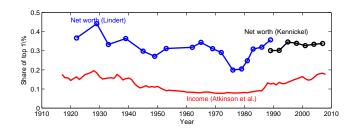


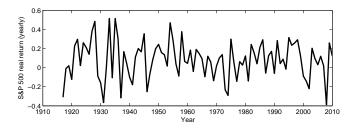
## Inequality and 5-year stock returns





## Inequality and stock-market volatility







# Extra: Advantages of continuous time

- **1** Agent only constrained when x = 0 (discrete time: also for some x > 0)
- **2** Can study **jumps** in asset returns at x = 0 analytically.
- 3 Euler equations give us differential equations for policies and prices.
  - In discrete time, have to go through all cases: Go broke in 1,2,3,... periods.
  - Have *N* initial and *M* terminal conditions: Tells us how many equilibria to expect (generically).



