Ricardian Equivalence Revisited: Deficits, Gifts and Bequests☆

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Abstract

Barro (1974) shows that operative altruistic transfer motives are key for Ricardian equivalence to hold. This paper evaluates the importance of this mechanism quantitatively by studying deficit-financed tax cut experiments. I use a heterogeneous-agents overlapping-generations economy with *endogenously* operative transfer motives to capture the fact that empirically transfers occur in some but not in all families. Altruism is calibrated to match aggregate transfer statistics. I find that the response of aggregate consumption to a tax cut is in the ballpark of a standard overlapping-generations economy, that is, an economy without altruistic family links. Welfare implications of this economy, however, move closer to a dynastic economy.

Keywords: Ricardian equivalence, consumption response to tax cut, interaction between government and private transfers.
JEL Codes: D64, H31, H62.

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1 Introduction

Macroeconomists extensively use infinitely-lived agent models. One very influential justification for this model is Barro's result: An economy with finitely-lived individuals (e.g. overlapping generations) is equivalent to an infinitely-lived agent economy if altruistic transfer motives are *universally* operative, that is, transfers are an interior solution for all current and future generations. This mechanism creates a chain linking all households in a dynasty, and so any individual household's planning horizon is extended to an infinite time horizon.

A stark policy implication is the irrelevance of government-financing schemes for the economy's allocation (*Ricardian equivalence*). A substitution of budget deficits for current taxation, a practice many governments have been engaging in, creates neither winners nor losers. Individuals confronted with low taxes take into account that taxes must increase in the future. In anticipation, they accumulate wealth, just enough to undo government borrowing, and if they themselves do not have to pay the increased tax burden, pass on their wealth to future generations. An absence of these household linkages, to the contrary, increases aggregate consumption of current generations at the expense of future ones.

Empirically these intergenerational linkages then imply transfers in the form of gifts (inter-vivos transfers) and bequests.[1] While these do not have to be large for Barro's mechanism to work they have to be common (Barro, 1989). Thus, concluding from the observation that because gifts and bequests do not occur in all families implies Barro's neutralization channel is of no consequence, is premature. Instead, assessing its importance is a quantitative question and requires a model. Since transfers are the vehicle of Barro's mechanism, such a model should generate plausible transfer behavior.

I build a model with the following features. First, I model *heterogeneous* families consisting of young and old agents in order to have transfer motives that are operative in some families but not in others. Second, there are no state-contingent assets. Agents face idiosyncratic income risk and can self-insure through savings in a risk-free asset, subject to a no-borrowing constraint. This type of incomplete market structure follows the bulk of the quantitative heterogeneous-agents literature. Third, at any point in time, an agent can choose a non-negative gift to the other family member but cannot commit to future transfers. The

[1] One could explain the occurrence of transfers by motives other than Barro/Becker-type altruism, such as "warm glow " altruism, but would then exclude Barro's mechanism a priori. Also, the data suggests that the incidence of transfers *does* depend on the economic well-being of the recipient which is contrary to warm glow. Andreoni (1989) and Abel & Bernheim (1991), for example, explore aspects of fiscal policy using alternative motives.

second and the third modelling choice are essential to replicate stylized facts on inter-vivos transfers, see below. Old agents face a mortality hazard. Wealth left over after the old agent in the family dies is bequeathed to the young. Thus, in the absence of annuity markets bequests are partially accidental, consistent with bequest behavior in the data (see, for example, Laitner & Juster, 1996).

A key feature of the model is that intergenerational altruism, in the sense of Barro (1974) and Becker (1974), spans the range from zero altruism (standard OLG economy) to perfect altruism (infinitely-lived/dynastic economy). If altruism is high, then transfers flow in many families, and one would expect the response of aggregate consumption to a tax cut to be small (and, vice versa, for low altruism). I select the degrees of altruism among generations to make them consistent with aggregate transfer statistics.[2] Unsurprisingly, agents are found to be imperfectly altruistic.

A crucial feature of this imperfect-altruism (IA) economy is that it delivers intra-family wealth distributions and clear predictions on when transfers flow. This is not the case in models with commitment, as pointed out by Barczyk & Kredler (2014*b*). Furthermore, transfer behavior is qualitatively in line with micro-level data on inter-vivos transfers. In the IA economy, as in the data, transfers are increasing in the donor's income and wealth, decreasing in the recipient's income (see McGarry & Schoeni 1995, 1997 and Berry, 2008), and typically flow from wealth- and income-rich households to relatively poor, liquidity constrained ones (see Cox, 1990, Cox & Jappelli, 1990, and McGarry, 1999). Quantitatively, the model generates fractions of constrained households and inter-vivos recipients closer to the data.

I compare the IA economy to two commonly used special cases of altruism in macroeconomics. Perfect altruism yields an infinitely-lived household model (or dynastic model), and I will refer to this economy as the perfect-altruism (PA) economy. Note, however, that here Ricardian equivalence will not hold due to the presence of borrowing constraints.[3] The implication of perfect altruism is that agents in a family essentially pool their resources, and so what matters for decision-making and welfare are total family resources, but not how they are distributed across the members. Importantly, this also implies that the timing of transfers is indeterminate, which is problematic since in the data we see clear patterns, see

[2]Here I use data only on aggregate inter-vivos transfers, and not bequests, since these are intentional in nature and thus more appropriate to identify the degrees of altruism; bequests may or may not be intentional. The data is taken from Gale & Scholz (1994).

[3]Laitner (1992), Heathcote (2005), and Fuster et al. (2007), for example, study redistributive fiscal policy using the dynastic framework when borrowing constraints are present.

above.[4] At the other end of the spectrum, the IA economy turns into a standard OLG economy (without family linkages) when there is no altruism, and I will refer to this economy as the no-altruism (NA) economy. In this economy, there are no intentional transfers, and hence Barro's mechanism is excluded a priori.[5]

I study deficit-financed tax cuts to address the following questions: What is the response of aggregate consumption? How do private and government transfers interact? What are the welfare implications of such policies?

I find that the aggregate consumption response in the IA economy is large, in the sense that it is similar to the NA economy. This is partially due to the fact that altruism identified through the data on aggregate transfers is far from perfect. Surprisingly, though, the response in the IA economy even *exceeds* the NA economy's, whereas, intuitively one would expect it to lie between that of the NA and PA economies. This intuition fails because the combination of imperfect altruism and no commitment imply strategic considerations in the consumption-savings decision.[6] An agent faces additional incentives to consume the increase in disposable income if the other family member accumulates additional savings since then she can expect higher transfers in the future.

The large consumption response is also driven by the fact that there is a larger fraction of borrowing-constrained agents, which is much more in line with the data, than in either the NA or PA economies. This is because in this economy the utility cost from being constrained is lower due to the feature that transfers flow precisely when an agent is constrained. But, in the IA economy a larger fraction of constrained agents does not automatically imply a larger response in consumption to a tax cut. It also depends on how many of the constrained agents are gift recipients since a tax cut crowds-out private transfers. This is because a donor reduces gifts by the amount of the tax relief and so gifts are merely replaced by higher disposable income and consumption for gift recipients may not change at all.

In terms of welfare, the IA economy loses its resemblance to the NA economy and moves

[4]Indeed, the literature that needs to deal with family transfers often takes the stance that transfers can only occur when one of the households is constrained; see, for example, Laitner (2001), Nishiyama (2002) and Fuster et al. (2007).

[5]Diamond (1965), Auerbach & Kotlikoff (1987) and Kitao (2010) use the standard OLG framework to study fiscal policies.

[6]To see this, consider the following static game between players 1 and 2: in the first stage of the game the players decide on a non-negative transfer. In the second stage they consume what is left. Player 1 and 2 are endowed with wealth w_1 and w_2. With log-utility, for example, it is easy to see that transfers flow if $\alpha w_1 > w_2$, where α is the degree of altruism. In a dynamic framework, wealth is endogenous, and so strategic considerations arise. For an analysis of these strategic interactions in two-period models see, for example, Lindbeck & Weibull (1988) and Bernheim & Stark (1988).

closer to the PA economy. The reason has to do with the crowding out of private transfers. In the NA economy, a young poor agent usually gains from a tax cut: higher net income when young is roughly compensated by lower net income when old, but the tax cut relaxes borrowing constraints. In the IA economy a young agent can paradoxically lose from a tax cut because the government alters private transfer behavior. The old reduces the transfer to the young in response to the tax cut keeping the young's consumption constant. However, the young must pay back the accumulated debt with higher taxes in the future. Had there been no tax cut, on the other hand, no future obligation would have arisen (that is, the old does not expect the young to repay whereas the government does).

In terms of the modeling framework and the computational methods, I build on Barczyk & Kredler (2014*a*). They study computationally a dynamic game of two infinitely-lived imperfectly altruistic agents in an incomplete-market setting restricting attention to Markov-perfect equilibria and provide a numerical solution algorithm. Using the limit of finite-horizon games as the equilibrium selection criterion, they find that the unique equilibrium is one in which transfers are delayed until the recipient is borrowing constrained. This type of equilibrium is also obtained in my setting and I conjecture it to be unique based on this previous work.

However, in terms of substance my paper differs starkly. I provide a quantitative analysis of how government transfers interact with private transfers; this is not done in Barczyk & Kredler (2014*a*). I *calibrate* an *OLG economy* with imperfect altruism to study *aggregate outcomes*, instead of relying on a *numerical example* to understand interactions of two *infinitely-lived* players. I focus on the transition period of the counterfactual policy experiments and not on the stationary equilibrium: I study the size of the consumption response to a tax cut, the quantitative role of the crowding-out effect of tax cuts on private transfers, and the welfare implications. I place the IA economy in contrast to calibrated versions of the NA and PA economies, which are standard workhorse models in macroeconomics, a comparison which previous literature, to the best of my knowledge, has not done before.[7]

Substantively my paper is closer related to Laitner (1988), Laitner (1992) and Heathcote (2005). Laitner (1988) studies the subgame perfect equilibrium of a heterogeneous-agents OLG economy with imperfect altruism. He finds that transfers flow only for certain family

[7] Barczyk & Kredler (2014*b*) is a theoretical exercise that strips down the setting from Barczyk & Kredler (2014*a*) to a very stylized one. In their deterministic setting the type of equilibrium of the current paper, and the one from Barczyk & Kredler (2014*a*), does not exist. Instead, Barczyk & Kredler (2014*b*) construct a continuum of equilibria in which the two players act as if they were a perfectly-altruistic dynasty. These equilibria exhibit tragedy-of-the-commons type inefficiencies despite well-defined property rights.

lines, but has no quantitative results (see also Abel, 1987 for a theoretical characterization of when gift and bequest motives are operative). Additionally, Laitner's generations overlap for only one period, whereas here, generations overlap for many time periods. This opens up the study of a transition period, whereas Laitner focuses on long-run steady-states, and also allows for more realistic transfer behavior. Both Laitner (1992) and Heathcote (2005) use a dynastic model to quantify deviations from Ricardian equivalence. Thus, these papers assume perfect altruism.[8] My paper extends their results in the following ways. First, I provide a comparison to the NA and PA benchmarks, second, I zero in on Heathcote's measure of deviations from Ricardian equivalence and show how much of these deviations is due to changes in agents' policies and what is driven by changes to the distribution of agents over the state space. Third, in addition to studying the consumption response, I also provide a welfare analysis. My paper is also related to a sequence of papers by Altig & Davis (1988,1992,1993). They study theoretically the consequences of redistributive fiscal policies in the presence of financial-market frictions when altruism is imperfect. However, in their framework, commitment is present. As is the case for the dynastic model, the commitment assumption leads to a lack of predictions on when transfers occur.

Finally, the empirical literature on Ricardian equivalence provides little consensus on the consumption effects that deficits stimulate. Bernheim (1987), for example, argues that practically all studies indicate that a dollar of deficits stimulate between 20-50 cents of consumer spending. On impact, I find that aggregate consumption responds by between 10-47 cents for durations of the tax cut ranging from two to 25 years, see Table 6 in the appendix, which appears to be in the ballpark. However, one should be cautious when comparing these numbers. A difficulty that the empirical literature encounters when studying Ricardian equivalence is that changes in the timing of taxes cannot be studied in isolation due to confounding factors, such as, changes to government expenditures. In my experiments agents know with certainty that taxes have to be paid back and that government expenditures remain constant.

2 The Model

2.1 Setting I model a small open economy. Time t is continuous. There are two life-cycle stages, young (y) and old (o). A young and an old agent make up a family. At each point in time there is a large (measure one) number of young agents and a large (measure one)

[8]For papers which use the dynastic framework to study other types of changes to government policies see, for example, Fuster et al. (2003, 2007, 2008).

number of old agents. An old agent faces a mortality hazard given by Poisson rate δ. Upon the death of an old agent, the young agent in the family becomes old and is matched with a new young agent; in this way, a new family is formed.[9]

Individuals face an idiosyncratic labor-income process; aggregate uncertainty is absent. Idiosyncratic productivity ϵ follows a Poisson process[10] with states $\{\epsilon_1, \epsilon_2, \epsilon_3\}$, where $\epsilon_1 < \epsilon_2 < \epsilon_3$. Transitions from the low or the high realization to the middle one occur at rate σ and from the middle to either the low or the high one at rate $\sigma/2$. A young agent's labor endowment corresponds to the idiosyncratic productivity, $w^y = \epsilon^y$. An old agents's endowment is given by the function $w^o(\epsilon^o)$, which will be a weighted average of labor earnings and Social Security benefits. When a new young agent enters the economy, its initial labor productivity depends on the labor productivity of the agent it is matched with. I denote by $\pi(\tilde{\epsilon}|\epsilon)$ the probability that the new young agent has productivity $\tilde{\epsilon}$ given that the agent it is matched with has productivity ϵ. Over the life cycle, income shocks to old and young are independent. When the old agent in the family dies, left-over wealth is bequeathed to the young agent in the family.

The market arrangement follows standard incomplete-markets models. Markets to insure against idiosyncratic risks are absent. There is a single, risk-free asset that pays a rate of interest, r. A household can hold a non-negative amount, $a \geq 0$, of the asset.

Flow utility for a young agent is $u(c^y) + \alpha^y u(c^o)$ and for an old agent it is $u(c^o) + \alpha^o u(c^y)$, where $\alpha^y, \alpha^o \in [0, 1]$ are the respective degrees of altruism and u is a CRRA utility function. For the PA economy $\alpha^y = 1 = \alpha^o$ and in the NA economy $\alpha^y = 0 = \alpha^o$. For any other combination of altruism we are in the IA economy.

Decision-making is simultaneous. At each point in time t, an agent's actions are to choose consumption $c \geq 0$ and gifts $g \geq 0$ taking the actions of the other family member as given. These decisions induce laws of motion for wealth

$$\begin{aligned} \mathrm{d}a^y &= (\underbrace{ra^y + (1-\phi)w^y + g^o - c^y - g^y}_{=\dot{a}^y})\mathrm{d}t, \qquad (1) \\ \mathrm{d}a^o &= (\underbrace{ra^o + (1-\phi)w^o + g^y - c^o - g^o}_{=\dot{a}^o})\mathrm{d}t. \end{aligned}$$

[9] The way I model the life cycle has the advantage that it is computationally tractable and that it introduces few new parameters; there is only one hazard rate and age is not a state variable.

[10] A Poisson process is the continuous-time counterpart to a discrete-state Markov process and therefore models persistent earnings.

The savings rates are denoted by $\dot{a}^y$ and $\dot{a}^o$ and are residually determined. The labor income tax rate is denoted by ϕ. The laws of motion for wealth apply as long as there is no bequest; when the old dies and leaves a bequest there is a discrete change in wealth.[11] Strategies are measurable with respect to the payoff-relevant state which consist of the young agent's wealth a^y and productivity ϵ^y, the old agent's wealth a^o and productivity ϵ^o, and time t. In order to economize on notation, I denote the state of an agent by (t, x) where $x = (a^y, a^o, \epsilon^y, \epsilon^o)$. The state space, excluding time, is denoted by $X = \{(a^y, a^o, \epsilon_i^y, \epsilon_j^o) : a^y \geq 0, a^o \geq 0, (i, j) \in \{1, 2, 3\}\}$.

The best-response problem is characterized by Hamilton-Jacobi-Bellman equations (HJBs). Section A.2 provides the derivation of the HJBs. Denote by $V^y(t, x)$ and $V^o(t, x)$ the young and the old agent's value function, respectively. Given the old agent's strategy $\{c^o(t, x), g^o(t, x)\}$, the young agent's value function and its partial derivatives have to satisfy

$$\begin{aligned} \rho V^y = V_t^y + \max_{c^y \geq 0, g^y \geq 0} \{u(c^y) + \alpha^y u(c^o) + \dot{a}^y V_{a^y}^y + \dot{a}^o V_{a^o}^y\} + \\ + \sum_{j=1}^{3} e(i^y, j) V^y(\cdot, \epsilon^y = \epsilon_j) + \sum_{j=1}^{3} e(i^o, j) V^y(\cdot, \epsilon^o = \epsilon_j) + \delta \left(W - V^y\right), \end{aligned} \tag{2}$$

subject to (1), where $V_t^y, V_{a^y}^y$, and $V_{a^o}^y$ are partial derivatives of the young's value function with respect to time t, own wealth a^y, and wealth of the old agent a^o. The transition rates of labor productivity from current state $i \in \{1, 2, 3\}$ to state $j \in \{1, 2, 3\}$ are summarized by hazard matrix

$$e = \begin{bmatrix} -\sigma & \sigma & 0 \\ \frac{\sigma}{2} & -\sigma & \frac{\sigma}{2} \\ 0 & \sigma & -\sigma \end{bmatrix}, \tag{3}$$

where $e(i, j)$ is the (i, j)th element of matrix e and i^y and i^o indexes the current productivity state of the young and the old agent, respectively. $V^y(\cdot, \epsilon^y = \epsilon_j)$ is the value function of the young agent if its own labor productivity is ϵ_j and $V^y(\cdot, \epsilon^o = \epsilon_j)$ is its value function if the old agent's productivity is ϵ_j.[12]

[11] Following Barczyk & Kredler (2014*a*) a noise term is added into the law of motion to ensure equilibrium existence; it's size matters little for the quantitative results presented later. For the sake of clarity I omit this technicality since it adds no additional economic intuition for the results of the paper.

[12] For example, if the young's current productivity is ϵ_2, and so $i^y = 2$, then

$$\sum_{j=1}^{3} h(i^y, j) V^y(\cdot, \epsilon^y = \epsilon_j) = \frac{\sigma}{2}[V^y(\cdot, \epsilon_1) - V^y(\cdot, \epsilon_2)] + \frac{\sigma}{2}[V^y(\cdot, \epsilon_3) - V^y(\cdot, \epsilon_2)].$$

When the old agent dies, the young becomes old, obtains a non-negative bequest, and is matched with a new young agent. These consequences are contained in the term $\delta(W - V^y)$. The function W is defined in the following way

$$W \equiv W(t, A, \epsilon) = \sum_{\tilde{\epsilon}} \pi(\tilde{\epsilon}|\epsilon) V^o(t, 0, A, \tilde{\epsilon}, \epsilon), \tag{4}$$

where $A = a^y + a^o$ is the sum of own savings, accumulated during life-cycle stage young, and the non-negative bequest a^o, and $\tilde{\epsilon}$ is the productivity realization of the new young agent. Its initial productivity realization depends on that of the new old agent, ϵ, through $\pi(\tilde{\epsilon}|\epsilon)$.

The old agent's HJB is almost mirror-symmetric to that of the young. Given the young's strategy $\{c^y(t,x), g^y(t,x)\}$, the old agent's value function and partial derivatives have to satisfy

$$\begin{aligned}\rho V^o = V_t^o + \max_{c^o \geq 0, g^o \geq 0} \{u(c^o) + \alpha^o u(c^y) + \dot{a}^o V_{a^o}^o + \dot{a}^y V_{a^y}^o\} + \\ + \sum_{j=1}^{3} e(i^o, j) V^o(\cdot, \epsilon^o = \epsilon_j) + \sum_{j=1}^{3} e(i^o, j) V^o(\cdot, \epsilon^y = \epsilon_j) + \delta(\alpha^o W - V^o),\end{aligned} \tag{5}$$

subject to (1) and transition rates given by (3). Note that the only qualitative difference in this HJB in contrast to HJB (2) is that W, given by equation (4), is weighted by the old's degree of altruism towards the young agent, α^o. Section A.1 in the appendix shows why this is the case.

Government consumption G is constant. It is financed through a proportional tax ϕ on labor income and by making use of deficits. The flow version of the government budget constraint is given by

$$\dot{D} = rD + G - \phi Y, \quad \text{given} \quad D_0, \tag{6}$$

where Y is the (time-invariant) aggregate gross endowment and D stands for government debt.

2.2 Optimality A crucial simplification of continuous time with respect to discrete time is that contemporaneous decision-making takes place without having to consider the actions of the other agent. Concretely, the young and the old agents' best-response consumption and

Thus, when labor-income uncertainty is a Poisson process the "expected value" shows up as the Poisson-weighted change in the value function of leaving the current state and entering a new one.

transfer functions are constant over an instant of time so that there is no need to calculate a best response for one agent for each action of the other agent as would be the case in discrete time. Since best-response functions are constant it also follows that when computing the game there is always a unique Nash equilibrium at each point in the state space.[13]

2.2.1 Static optimality Intra-temporal optimality is characterized by the same conditions for young and old agents and so I restrict the analysis here to old agents. Optimal consumption is determined by $\max_{c^o \geq 0}\{u(c^o) - c^o V^o_{a^o}\}$ with first-order condition $u_c(c^o) = V^o_{a^o}$. Marginal utility of current consumption is equal to the marginal value of saving. Crucially, the old agent does not have to contemplate contemporaneous actions by the young agent. Thus, over a short amount of time, the best-response consumption function is constant and can be obtained as in a typical consumption-savings problem.

The old agent's Lagrangian for transfers is $\mathcal{L} = g^o(V^o_{a^y} - V^o_{a^o}) + \mu^o g^o$ where μ^o is the multiplier on the constraint $g^o \geq 0$. The first-order condition with respect to g^o is $\mu^o = V^o_{a^o} - V^o_{a^y} \geq 0$. In any equilibrium this multiplier has to be non-negative. It is insightful to consider why. The multiplier measures the old's marginal value of taking one unit of resources away from the young and giving it to the old. The marginal gain to the old is $V^o_{a^o}$ and the marginal loss is $V^o_{a^y}$. Thus, when $\mu^o > 0$ the old agent desires a negative transfer, $g^o < 0$, which in the current environment is not feasible. If $\mu^o < 0$, however, the old sets a transfer which instead of a rate is a mass-point. Such a transfer induces a discrete jump in the state space and ensures that μ remains non-negative.[14] When $\mu^o = 0$ the old is locally indifferent with regard to the intra-family wealth distribution and so any transfer flow is consistent with optimality.

Only the PA economy has the feature that the multiplier equals zero everywhere. Agents of the same family are indifferent (locally and globally) with regards to the intra-family wealth distribution rendering transfers and the intra-family wealth distribution indeterminate. In contrast, in the equilibrium of the IA economy multipliers are strictly positive (a computational result) and gifts only flow when the recipient is borrowing constrained. Transfers are

[13]The technical reasons are that second-order effects vanish as time becomes continuous and that flow utility $u(c) + \alpha u(\tilde{c})$ is separable. For a more extensive discussion on the advantages of continuous time in a dynamic setting with imperfectly-altruistic players see Barczyk & Kredler (2014*b*).

[14]In the equilibrium gifts are rates, just like consumption and savings, and are thus a flow, i.e. over a short period of time Δt the flow of transfers is $g\Delta t$. A mass-point transfer instantly induces a transfer of large quantity (mass) $g \in [0, a]$ instead of a flow. This type of transfer is like a bequest. Of course, bequests do occur in the equilibrium but not mass-point gifts. It is always to the advantage of the donor to delay transfers for as long possible, which is, until the recipient is constrained. Transfers occur then only as rates.

then determined as follows.[15] Suppose the young agent is constrained, $u_c(w^y) > V^y_{a^y}$, the old is unconstrained, $V^o_{a^o} = u_c(c^o)$, and transfers are zero, $g^o = 0$. Since transfers are zero, it must be the case that $u_c(c^o) > \alpha^o u_c(w^y)$. If not, the old agent would choose a positive amount of transfers in order to equalize her consumption and gift margins, $V^o_{a^o} = u_c(c^o) = \alpha^o u_c(w^y + g^o)$. The old can then "dictate" the young's consumption, i.e. $c^y = w^y + g^o$.

The economic intuition of why gifts in the IA economy are delayed is as follows. An imperfect altruist does not fully internalize the effect its consumption behavior has on family resources. Thus, an *early* transfer would be consumed at a faster rate than the donor desires. After consuming the transfer, the recipient would come back and ask for more – after all, the recipient knows that the donor cannot credibly commit to not provide a transfer again. However, when an agent is constrained a donor can exert control over the recipient's consumption behavior, temporarily implementing his preferred allocation.

2.2.2 Dynamic optimality I now highlight crucial differences between the various economies by way of the Euler equations.[16] Section A.3 in the appendix provides a derivation of the Euler equation for the IA economy.

The Euler equations for unconstrained old and young agents are, respectively,

$$\mathcal{A}u_c(c^o) = \underbrace{(\rho - r)u_c(c^o)}_{\text{standard}} + \underbrace{\left[V^o_{a^y} - \alpha^o u_c(c^y)\right]c^y_{a^o}}_{\text{altruistic-strategic distortion}} - \delta \underbrace{\left[\alpha^o W_A - u_c(c^o)\right]}_{\text{bequest}}, \tag{7}$$

$$\mathcal{A}u_c(c^y) = (\rho - r)u_c(c^y) + \left[V^y_{a^o} - \alpha^y u_c(c^o)\right] c^o_{a^y} - \delta \underbrace{\left[W_A - u_c(c^y)\right]}_{\text{aging}},$$

where the operator $\mathcal{A}$ is the "expected time derivative"

$$\mathcal{A}V_a(t, x_t) \equiv \lim_{\Delta t \to 0} \frac{1}{\Delta t} E_t[V_a(t + \Delta t, x_{t+\Delta t}) - V_a(t, x_t)].$$

$c^y_{a^o}$ and $c^o_{a^y}$ are partial derivatives of consumption policies with respect to wealth of the other family member and measures how consumption responds to changes in the wealth position of the other agent. Thus, the Euler equations explicitly draw attention to the strategic considerations present in the IA economy while the HJBs contain them only implicitly in the value functions. As with any other Euler equation, when dividing by the marginal utility of

[15] The constrained case has more cases than presented here, but this simple example conveys the intuition well. For an analysis of *all* cases that can arise see Section 3.3 in Barczyk & Kredler (2014*a*).

[16] Barczyk & Kredler (2014a), in Section 3.2, analyze the Euler equation of infinitely-lived agents. Here there is an additional consumption-savings trade-off due to death and aging.

consumption the terms on the right-hand side tell us the expected growth rate of marginal utility.

In order to see what this equation says, suppose old agent o contemplates a deviation from the current equilibrium path by temporarily increasing wealth before returning to the equilibrium path. There is the standard trade-off between lower current and higher future consumption given by the term $(\rho - r)u_c(c^o)$. But now there is the additional consideration that the young will change its consumption. If $c^y_{a^o} > 0$[17] agent o knows that in response to an increase in wealth consumption of agent y will also increase. On the one hand, this yields utility $\alpha^o u_c(c^y)c^y_{a^o}$ to o and thus constitutes an additional benefit from saving since it enters the Euler equation with the same sign as the interest rate does. On the other hand, y's consumption reduces its wealth a^y and so a new equilibrium path is entered in which agent y has $c^y_{a^0}$ less wealth. This poses an extra cost to o given by $V^o_{a^y}c^y_{a^o}$ which enters the Euler equation with the same sign as does ρ. When the term altruistic-strategic distortion is positive, agent o consumes at a rate higher than in the absence of a strategic motive. Computationally, this term tends to be positive and so acts to increase impatience and induces the agent to over-consume.

The aging-term can be interpreted as a precautionary savings motive for the young agent. Upon the death of the old agent, the young becomes old, owns wealth A, and is matched with a new young household of uncertain labor productivity. The young's expected marginal utility of consumption given the old agent's death is $W_A = \mathbb{E}(u_c(c^o_t)|\text{death}_t)$.[18] If $\mathbb{E}(u_c(c^o_t)|\text{death}_t) > u_c(c^y_t)$, aging creates an incentive to save. For the old agent, the counterpart term constitutes a motive to save for bequests. If $\alpha^o W_A > u_c(c^o)$ the old has an additional incentive to save because she places a higher value of leaving an additional unit of wealth as a bequest than from consuming it herself.

The Euler equation for the NA economy is a special case of equation (7) and is obtained when setting $\alpha^y = 0 = \alpha^o$. For the old agent, the payoff relevant state consists of (t, a^o, ϵ^o) and her Euler equation is

$$\frac{\mathcal{A}u_c(c^o)}{u_c(c^o)} = (\rho + \delta) - r. \tag{8}$$

As in Yaari (1965), lifetime uncertainty enlarges the discount rate to $\rho + \delta$ by the rate of

[17]This is reasonable since a higher level of wealth for one agent implies that transfers are more likely and larger and the likelihood that the other agent has to provide transfers in the future is reduced.

[18]To see this, take the derivative of W with respect to A in equation (2) to get $W_A = \sum_{\tilde{\epsilon}} \pi(\tilde{\epsilon}|\epsilon^o = \epsilon^y)V^o_A(\tilde{\epsilon}, \cdot) = \sum_{\tilde{\epsilon}} \pi(\tilde{\epsilon}|\epsilon^o = \epsilon^y)u_c(c^o(\tilde{\epsilon}, \cdot)) = \mathbb{E}(u_c(c^o_t)|\text{death}_t)$.

mortality. For the young agent, however, the old agent's economic characteristics continue to be part of her payoff-relevant state due to the presence of accidental bequests. Strategic considerations, however, are absent since $c^o_{a^y} = 0$. The young agent's Euler equation is

$$\frac{\mathcal{A}u_c(c^y)}{u_c(c^y)} = \rho + \delta\left(1 - \frac{u_c(c^o)}{u_c(c^y)}\right) - r, \tag{9}$$

In order to interpret this equation, consider two identical young agents except that only one can expect a bequest. Then, $u_c(c^o(\cdot, a^y)) > u_c(c^o(\cdot, a^y + a^o))$ and so the agent that cannot expect a bequest faces a larger incentive to save.

In the PA economy, $\alpha^y = 1 = \alpha^o$ and so individual agents' decision-problems can be pooled into a single joint-maximization problem. The payoff-relevant state consists of the combined level of wealth $a = a^y + a^o$ and the productivity profile (ϵ^y, ϵ^o). Consumption and savings decisions depend only on total family resources and not on the intra-family wealth distribution. A PA family has one Euler equation given by

$$\frac{\mathcal{A}u_c(c)}{u_c(c)} = \rho + \delta\left(1 - \frac{W^{PA}_A}{u_c(c)}\right) - r, \tag{10}$$

where $W^{PA}(t, a, \epsilon) = \sum_{\tilde{\epsilon}} \pi(\tilde{\epsilon}|\epsilon) V^{PA}(t, a, \tilde{\epsilon}, \epsilon)$ and V^{PA} is the value function of the PA economy. If the setting were such that a family is matched with a new young agent of known productivity upon the death of the old agent, then $W^{PA} = u_c(c)$ and we would get the familiar Euler equation where the growth rate of marginal utility equals $\rho - r$. Here this is only approximately the case due to the assumption that a new young agent's productivity is not deterministic when entering the economy.

2.3 Equilibrium definition A *recursive Markov-perfect equilibrium* is given by a set of functions for young agents, $\{c^y(t,x), g^y(t,x), V^y(t,x)\}$, a set of functions for old agents, $\{c^o(t,x), g^o(t,x), V^o(t,x)\}$, and a measure $\lambda(t,x)$ of agents over time and the state space X, such that, given the interest rate, r, government policy rules, $\{\phi, D, G\}$, and an initial measure, $\lambda(0,x)$, the following hold

1. $\{c^y(t,x), g^y(t,x), V^y(t,x)\}$ solves the young agent's best-response problem, HJB (2) subject to (1), taking as given the policy functions of the old agent $\{c^o(t,x), g^o(t,x)\}$.

2. $\{c^o(t,x), g^o(t,x), V^o(t,x)\}$ solves the old agent's best-response problem, HJB (5) subject to (1), taking as given the policy functions of the young agent $\{c^y(t,x), g^y(t,x)\}$.

3. The government budget constraint, Equation (6), holds.

4. The measure λ satisfies

$$\int_B \lambda(t+\Delta t, x)\mathrm{d}x = \int_X \lambda(t,x)\Big[\int_B F(t,x,\tilde{x},\Delta t)\mathrm{d}\tilde{x}\Big]\mathrm{d}x, \quad \forall t, \forall \Delta t > 0$$

where

$$B = \{(a^y, a^o, \epsilon_i^y, \epsilon_j^o) : a^y \in [b_1, b_2], a^o \in [b_3, b_4], (i,j) \in \{1,2,3\}\},$$

where $\quad b_2 \geq b_1 \geq 0, \quad b_4 \geq b_3 \geq 0,$

and F is the transition density from x to $\tilde{x}$ over the time interval $[t, t+\Delta t]$ induced by the laws of motion for (a^y, a^o), Equation (1), shocks to labor productivity (ϵ^y, ϵ^o), given in (3), and mortality hazard δ.

3 Calibration

The three economies are calibrated to match the same wealth-to-GDP ratio by calibrating in each economy the rate-of-time-preference ρ. I do this to ensure that growth rates of marginal utility are roughly constant across the economies. Differences across the economies can then be more readily attributed to differences in the degrees of altruism and not to varying degrees of impatience. In order to see that impatience is different from the rate-of-time-preference ρ it is easiest to consider the Euler equations for the NA and PA economies in the presence of only mortality hazard. The growth rate of marginal utility in the NA economy is then $\rho + \delta$ and it is ρ in the PA economy. Thus, without re-calibrating ρ, the economies would generate differences in aggregate savings behavior simply due to different degrees of impatience, making a comparison based on altruism challenging.

A further reason that necessitates a re-calibration concerns the existence of a stationary equilibrium. In order for a stationary equilibrium in the IA economy to exist, it has to be the case that impatience $> r$. If altruistic-strategic distortions are relatively large, see Equation (7), the calibrated value for ρ in the IA economy may have to be below the interest rate in order for agents to have enough of an incentive to save so that the economy produces the empirical wealth-to-GDP ratio. This is indeed the case in the preferred calibration. If we now use the calibrated value ρ of the IA economy for the PA economy, then $\rho < r$ and there will be no stationary equilibrium in the PA economy.

In order to calibrate the degrees of altruism α^y and α^o in the IA economy, the economy is calibrated to match empirical gift-to-wealth ratios of young and old. Separate identification of these parameters is straightforward: increasing the degree of altruism of the old leads to more young agents obtaining transfers and to increase the size of transfers, both of which increase the old's gift-to-wealth ratio, and, vice versa, for the young's degree of altruism.

3.1 Calibrating the stationary economy Section B.1 in the appendix defines the numerical algorithm used to compute the stationary equilibrium. Table 1 summarizes the pertinent features and parameters of the calibration. All flow variables are on a yearly basis.

The per-period utility function is given by

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma},$$

where γ is the coefficient of relative risk aversion. It is identical for both old and young agents with a coefficient of relative risk aversion equal to 2. The interest rate r is assumed to be 4%.

The life-cycle stages young and old correspond approximately to ages 25-50 and to ages 50-75, respectively. Therefore, the expected duration of a life-cycle stage is 25 years, implying a value of 4% for the mortality hazard δ.

The labor-income process is calibrated to the U.S. income distribution for households of ages 25-65. Only one rate parameter, σ, needs to be calibrated. It is chosen in order to match an annual auto-correlation for the labor-income process of 0.9.

Recall, $\pi(\tilde{\epsilon}|\epsilon)$ is the probability that a young agent enters the economy with income realization $\tilde{\epsilon}$, given that the new old agent has income realization ϵ. Analogously to the labor-income process, if the old agent is in a low or in a high income state, a young agent is assumed to be equally likely to enter the economy with the medium-level productivity realization. If the old agent is in the medium productivity state, the new young agent is assumed to enter the economy with a low or a high level of productivity with equal probability. Thus only one parameter needs to be calibrated. This parameter, $\hat{\pi}$, is chosen in order to match an intergenerational earnings elasticity of 0.4; see Solon (1999).

The income for an old agent (not shown in the Table) consists of a weighted average of the realization of the labor-income process and its corresponding Social Security income: $w^o(\epsilon^o) = \epsilon^o(\frac{3}{5} + \frac{2}{5}\kappa(\epsilon^o))$, where κ is the Social Security replacement rate and depends on the realization of the productivity shock; $\kappa_i = \{68\%, 50\%, 40\%\}$ for $i \in \{1, 2, 3\}$, where the replacement rates are taken from Mitchell & Phillips (2006). The weights $3/5$ and $2/5$

Table 1: Calibration of stationary economy

(a) common across economies

γ	r	δ	σ	$\hat{\pi}$	g	$\bar{D}$	$\bar{\phi}$
2	4%	4%	10%	0.36	20%	0	22.7%

Parameters three economies have in common. The coefficient of relative risk aversion is γ, the interest rate is r, mortality hazard is δ, the Poisson rate of earnings is σ, heritability-of-earnings probability is $\hat{\pi}$, the fraction of government purchases of GDP is g, the debt level is $\bar{D}$, and $\bar{\phi}$ is the tax rate to balance the government budget.

(b) specific to economy

	data	IA	NA	PA
calibration target				
wealth/GDP	3.0	3.0	3.0	3.0
old gift/wealth	0.32%	0.32%		
young gift/wealth	0.03%	0.03%		
parameter				
ρ		3.42%	3.29%	4.92%
α^y		0.110		
α^o		0.295		
not targeted				
constrained households	19.0%	19.4%	9.9%	6.2%
wealth-poorest 40%	3.1%	3.0%	5.7%	9.9%
young gift-recipients	16.1%	15.3%		
old gift-recipients	5.4%	2.6%		
bequest/wealth	1.06%	2.5%	2.3%	2.0%

Wealth-to-GDP ratio taken from Huggett (1996). Gift-to-wealth ratios and the bequest-to-wealth ratio are based on the 1983-86 Survey of Consumers Finances (SCF), Gale & Scholz (1994). Fraction of constrained households from 1983 SCF taken from Jappelli (1990): 12.5% of households have a request for credit rejected and a further 6.5% does not apply for it, expecting to be rejected. Wealth held by poorest 40% based on 1983 SCF, Wolff (1987). Fraction of old and young gifts recipients are based on Health and Retirement Study (author's own calculation).

capture the average duration an old agent spends earning labor income and collecting Social Security benefits, respectively.

Government expenditures are constant at 20% of stationary aggregate output. The debt level in the stationary equilibrium is assumed to be zero. The tax rate required to balance the government budget in the stationary equilibrium is 22.7%.

The aggregate transfer statistics used to identify the altruism parameters are from Gale & Scholz (1994). The authors document that the annual flow of *intended* transfers as a percentage of aggregate net worth is 0.53%. It is made up of 0.35% of support given to adult family members, 0.12% of trusts, and 0.05% of life insurance.[19] The 0.35% of support, as a percentage of aggregate net worth, given to adult family members is the appropriate counterpart in the data to the flow of annual gifts, as a percentage of aggregate net worth, generated by the IA economy. College expenses (0.29%) are excluded due to controversy whether they constitute a transfer. In order to separately identify the young's and the old's degree of altruism, I make use of the fact that the 0.35% consists of 0.32% of support given from old to young agents and of 0.03% of support given from young to old agents.[20]

The calibrated magnitudes of altruism are imperfect and asymmetric. The asymmetry is driven by the fact that the transfer-to-wealth ratio of the old generation is larger than that of the young. Note, however, while the gift-to-wealth ratio of the young is only one-tenth of the old's, young's altruism is still relatively sizeable. This is because old agents are richer than young ones and are less likely to be constrained. Altruism is far from perfect because observed aggregate transfers do not justify larger values. [21]

The calibrated values of the rate-of-time preference in the NA and IA economies are fairly similar and substantially lower than the one of the PA economy. The reason the PA economy needs a larger value for the rate-of-time preference is because of agents' concern for future generations. Mortality hazard plays a much smaller role in their consumption-savings choice, see Euler equation (10), and so agents in this economy are more patient. Thus, ρ has to be comparatively large; note, there would not exist a stationary equilibrium in the PA economy when using the calibrated value of ρ from the IA economy. In the NA economy ρ is smallest. Here, agents have no concern for future generations and so mortality

[19]The numbers on aggregate transfers appear to be very small because they are reported as an annual flow. Converting this flow into a stock, Gale & Scholz (1994) argue that intended transfers are the source of at least 20% of aggregate net worth.

[20]Nishiyama (2002) also utilizes these data for calibrating the degree of altruism of the old and uses a four-period OLG economy to study the concentration of the U.S. wealth distribution.

[21]For an economic interpretation of what the size of altruism means and how it depends on the size of risk aversion, see Section B.2 of the appendix.

hazard matters strongly for them. In order to have NA agents sufficiently patient, to achieve the targeted wealth-to-GDP ratio, their rate-of-time preference has to be low. The reason the IA economy's rate-of-time preferences is slightly above the NA economy's is that here agents care somewhat about future generations, which tends to make them more patient, but strategic considerations tends to make them less patient, see Euler equation (7).

I now turn to some pertinent moments not directly targeted by the calibration. The fraction of constrained agents in the IA economy is closest to the one in the data. Altruistic transfers help to generate these while the model is simultaneously able to produce enough wealth in the economy to obtain the targeted wealth-to-GDP ratio. The fraction of constraint agents is made up of 14.9% young and 4.5% old agents. The NA economy encounters difficulties in attaining the right wealth-to-GDP ratio and simultaneously generating enough poor agents. In this economy there are 6.9% young constrained agents and 3.0% old ones. The PA economy produces the smallest fraction of constrained agents. Here, income is pooled among family members which makes being constrained less likely. Similarly, in the IA economy the fraction of wealth held by the poorest 40% is closest to the one found in the data.

The IA economy is targeted to aggregate transfer statistics based on the SCF but the resulting fractions of gift recipients is in line with transfer data from the Health and Retirement Study. There are, however, too few old gift recipients in the model which may be due to underreporting of transfers in the SCF.

A shortcoming of the IA economy is that it generates a bequest-to-wealth ratio that is too large. This is likely due to the assumption that elderly in the model face a constant life expectancy. Also, note that the bequest-to-wealth ratio in the PA economy is 2% by construction: wealth, a, is commonly owned and so the bequest-to-wealth ratio is $\delta(0.5a)/a = 2\%$.

3.2 Set-up for tax-cut experiments Section B.1 in the appendix defines the numerical algorithm used to compute the transition path for the counterfactual experiments. I now describe the broad features of the changes in government-financing policy I consider.

Government expenditure is taken as given. In the stationary equilibrium these are financed only with taxes and so the government runs a balanced budget. The government announces a reduction in taxes and borrows the shortfall (from abroad if necessary), thereby increasing national debt. Eventually, taxes have to increase in order to reduce the debt burden. Once the debt is back to its steady-state value (assumed to be zero), the government returns to a balanced budget indefinitely. The present value of the tax burden is the same as it would have been under a balanced budget.

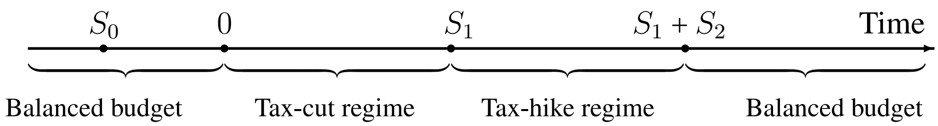


Figure 1: Timing of deficit-financed tax cut

Figure 1 illustrates the timing of a generic deficit-financed tax cut. Prior to S_0, the economy is assumed to be in the stationary equilibrium. The change in tax policy is announced at time S_0 and implemented at time 0. The reason the announcement and implementation do not happen concurrently is because in reality there is a lag between these two events. Furthermore, pre-announcing the new financing policy allows me to disentangle the effects which are due to information (*announcement effect*) versus the actual pay-out of the tax cut (*impact effect*). The tax cut lasts for S_1 years. The tax hike implemented afterwards is such that the new debt is paid-off over the following S_2 years. Afterwards, the tax rate returns to its steady-state value.

I impose the following structure on all experiments. The tax-cut regime lasts as long as the tax-hike regime, i.e. $S_1 = S_2$. A policy is announced one year prior to its implementation ($S_0 = -1$). Deficits finance 3% of stationary government consumption. This choice ensures that the size of the debt-to-GDP ratio remains reasonable for longer durations of the experiment. During either regime the tax rate is constant and debt is residually determined. I consider four durations. In the short-term experiment, there is a 2-year tax cut followed by a 2-year tax hike. The next is a medium-term one, corresponding to a 4-year tax cut and a 4-year tax hike. One interpretation of the short- and medium-term experiment durations is to think of them as corresponding to the U.S. presidential election cycle. I also study a full-generation-term experiment, with a 25-year tax cut and a 25-year tax hike. This experiment mimics the expected length of time young and old generations overlap from the inauguration of the policy. Finally, an experiment is included with a 12-year tax cut followed by a 12-year tax hike, which I call half-generation experiment, to have a case that lies between the short-term and the full-generation-term experiments.

4 Policy experiments

In order to gauge deviations from Ricardian equivalence I focus on the response of aggregate consumption to a deficit-financed tax cut. I construct a measure, based on Heathcote (2005), referred to as the *propensity to consume out of income taxes (PCT)*, see Section A.4 in the appendix for the exact construction of this measure,

$$\mathrm{PCT}_t \equiv \frac{\Delta C_t}{\Delta Y_t^d},$$

where ΔC_t is the difference between time-t and steady-state aggregate consumption, and ΔY_t^d denotes the difference between time-t and stationary aggregate disposable income. Thus, the PCT tells us the fraction consumed out of a \$1 tax cut (or hike). Under Ricardian equivalence (RE) agents' consumption remains unchanged and so the PCT equals zero. If all agents are hand-to-mouth consumers the entire tax cut is consumed and the PCT equals one. Figure 2 illustrates the economies' PCTs over time for all the experiments.

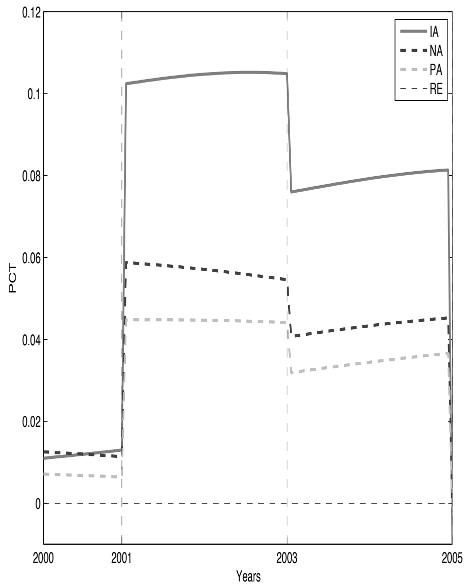
4.1 Announcement effect The government announces the change in financing policy in the year 2000. This new information has the following effect: the PCT jumps up in each of the economies. The initial jump and the ensuing trajectory of the PCT until 2001 are entirely driven by unconstrained agents as they want to ensure a smooth consumption path in anticipation of higher future consumption.[22] Constrained agents would also like to do so, but cannot, since the actual implementation of the tax cut takes place later.

For the time period 2000 to 2001, compare the PCTs across the experiments. Initial responses and ensuing paths are increasing in the duration of the deficit-financed tax cut. The reason is that the farther the tax hike lies in the future the more the tax cut becomes a permanent increase in income. In the IA and NA economies the initial responses are strongest because agents are least concerned about future generations. The IA economy is similar to the NA economy especially for the short- and the medium-term experiments.

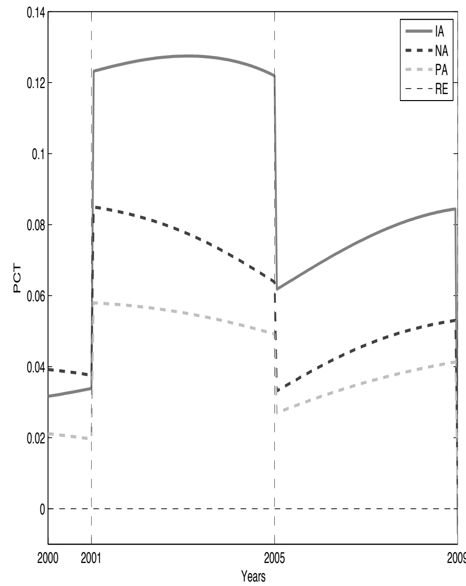
Unconstrained agents in the PA economy also increasingly react to the new information as the experiment lengthens. The main reason these agents are affected is due to the presence of borrowing constraints and income risk. Taken together, these features effectively shorten the planning horizon. Income uncertainty makes agents expect to be constrained at some point in the future and economic consequences beyond the time of a binding borrowing

[22] In order to compute the PCT for the period between the announcement and the implementation, I use the change in aggregate disposable income of the tax-cut regime.

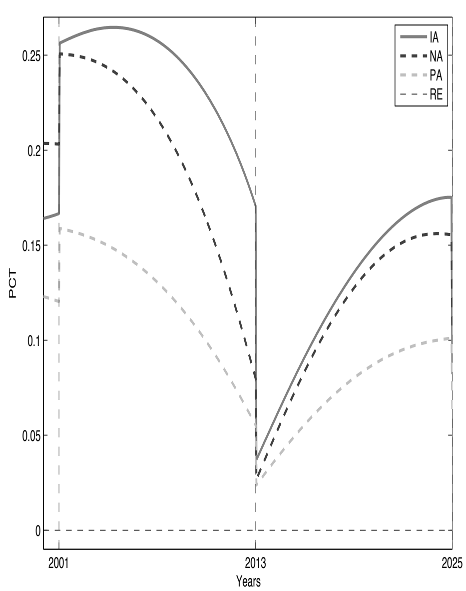
Figure 2: PCT over time



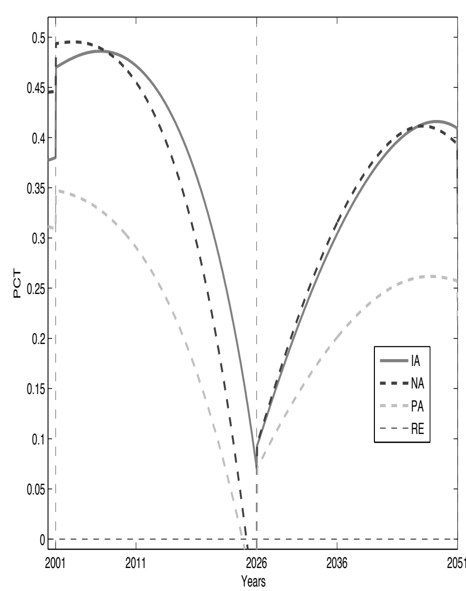
(a) short



(b) medium



(c) half-generation



(d) full-generation

The first and the second dashed vertical lines mark the onset of the tax-cut regime and the tax-hike regime, respectively. The horizontal dashed lines at zero is the PCT over time when RE holds.

constraint are not internalized. When considering the full-generation experiment we see that this effect undermines altruistic-linkages to a substantial extent. In this experiment, the announcement effect in the PA economy is about 32 cents compared to 45 cents in the NA economy in which altruism is entirely absent.

4.2 Impact effect The tax cut is implemented in 2001. Once again, the PCT jumps up, but this time solely due to constrained agents. Constrained agents in the NA and PA economies that remain constrained after receiving the tax cut consume the tax cut one-for-one.

Table 2: Impact effect

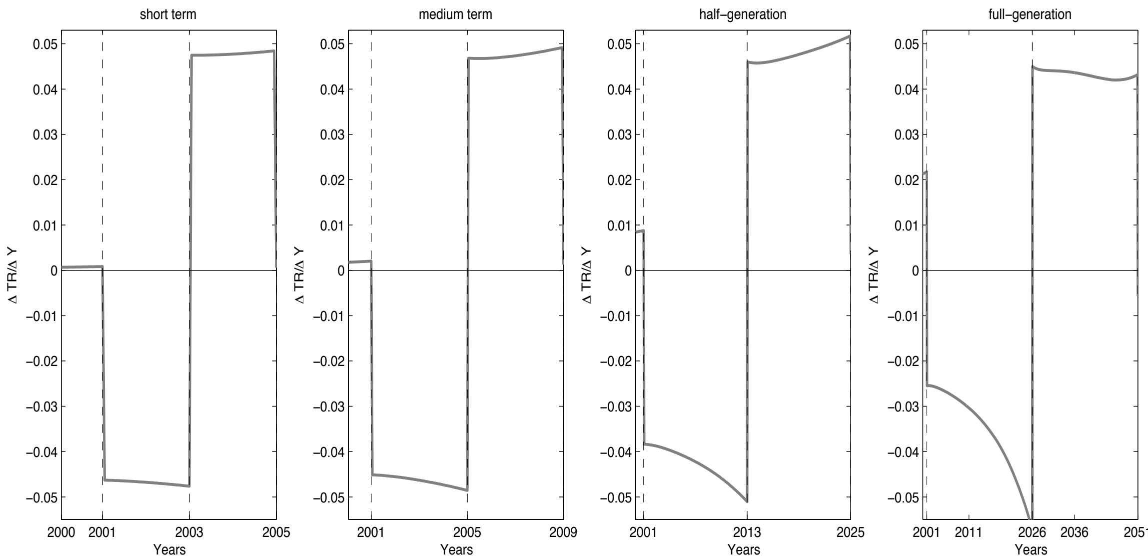
	NA	IA	PA
constrained	9.8%	19.5%	6.2%
jump in PCT	4.7	8.9	3.8

Fraction of constrained agents and jump in PCT at time of the tax cut in cents.

Table 2 puts the fraction of constrained agents and the jump in PCT in terms of cents when the tax cut is implemented into contrast. For the NA and PA economies, the impact effect equals the constrained agents' labor-income share; the aggregate PCT is given by $l \cdot \text{PCT}_{cstr} + (1 - l) \cdot \text{PCT}_{uncstr}$, where l is the labor-income share of constrained agents. At the time of impact, only constrained agents matter and since their PCT equals one the impact effect equals their labor share. Their labor-income share is below the population share of constrained agents because they tend to be income poorer than unconstrained agents. In the IA economy this is not the case for all constrained agents as there is also a *crowding-out* effect.

4.3 Crowding-out effect In the IA economy deficits crowd-out gifts and thus there are constrained agents that have a PCT of zero at the time of impact. Recall, a donor's first-order condition is $u_c(c) = \alpha u_c(g + (1 - \phi)w)$, where $(1 - \phi)w$ is the after-tax income of the transfer recipient. Suppose disposable income of the constrained transfer recipient increases by \$1. If prior to the tax cut the gift was \$1 or more, it is now optimal for the donor to reduce the gift amount by this tax cut and so consumption of the constrained agent remains unchanged. Only if the gift was smaller than the tax cut does consumption increase by the difference between the tax cut and the amount of the previously received gift.

Figure 3: Propensity to transfer out of income taxes



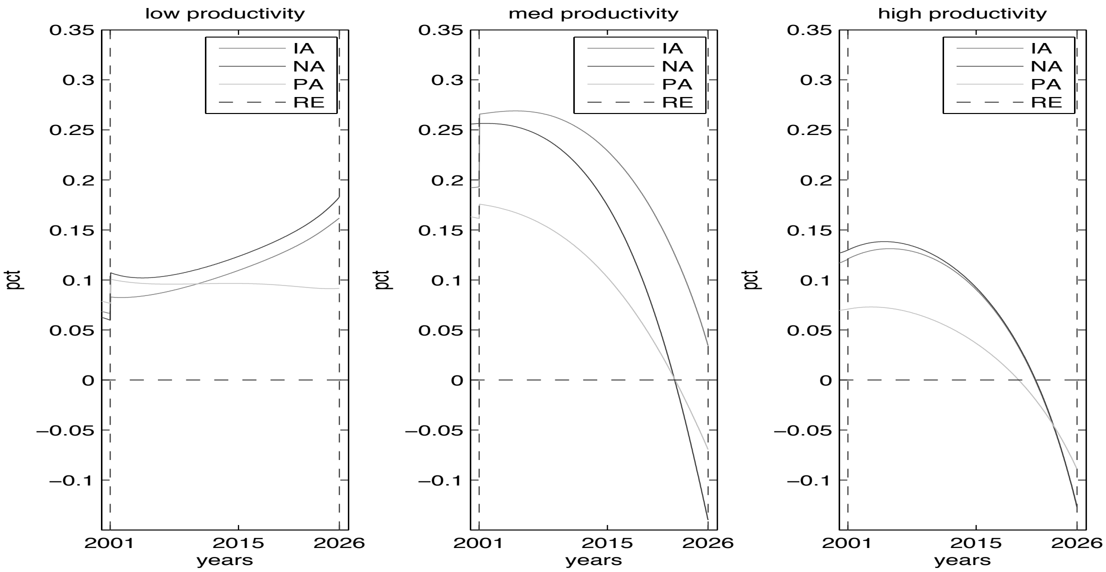
Change in aggregate gifts in terms of cents when aggregate disposable income changes by $1. Calculated in the same way as the PCT, except that aggregate consumption is replaced by aggregate transfers.

Figure 3 shows the propensity to transfer out of income taxes, that is, $\Delta TR_t/|\Delta Y_t^d|$, where ΔTR_t is the difference between time-t and steady-state aggregate gifts. There is a small announcement effect which increases in the duration of the experiment. This reflects the announcement effect from the unconstrained agents discussed above: as they increase consumption they provide higher gifts in accordance with their consumption-gift margin. At the impact stage aggregate transfers decrease as just explained. In the absence of this withdrawal of transfers, holding other things constant, we can conclude that the impact effect would be about 5 cents higher in the IA economy.

Over time, there is the additional effect that the fraction of gift recipients gradually decreases. This is because some gift recipients begin to save at which point they no longer obtain a gift. This is particularly visible in the full-generation length experiment.

4.4 Over-consumption effect In the IA economy the PCT exhibits the striking feature that it decreases at a much slower rate than in the other two economies. This feature is especially obvious in the full-generation length experiment in which the trajectory of the PCT starts out below the one from the NA economy but eventually surpasses it. The reason behind this shape has to do with strategic interactions, highlighted by the Euler equations (7), which are only present in this economy.

Figure 4: PCT by productivity



Full-generation length experiment. PCT for different productivity types for duration of tax-cut regime.

The economic intuition is that the possibility of future transfers decreases the marginal value of saving for an unconstrained agent if the other agent in the family starts to save more. The poorer household anticipates that future gifts or higher bequests are more likely and thus can effectively free ride on the frugal behavior of the family member. Additionally, by not engaging in additional saving itself the poorer agent can actually manipulate the frugal agent to save even more. As the economy approaches the date of the tax increase, it becomes more and more certain that current agents will carry the burden of paying back the accumulated debt through higher taxes. In the IA economy agents can rely on family members on being bailed out with gifts when taxes increase. Without altruism, as well as with perfect altruism, there is no such moral hazard and therefore savings increase by more.[23]

4.5 PCT by productivity Figure 4 shows the PCT for the full-generation experiment, over the duration of the tax-cut regime, separately for the different productivity types.

When considering the PCT for agents of low productivity we see that the impact and announcement effects together are stronger in the NA and PA economies than in the IA

[23]In the NA economy the anticipation of higher future bequests also implies that the incentive to save decreases as the old family member increases wealth, see the young agent's Euler equation (9). Here, however, an agent cannot manipulate the family member's behavior, because bequests are accidental, and so this is not a strategic consideration.

economy. This is explained by the crowding-out effect of private transfers by the tax cut and emphasizes the fact that in this economy the consumption response does not only depend on the fraction of constrained individuals but also how many receive private transfers. For individuals of medium productivity, the crowding out of transfers plays a less important role, and we can see that this group drives the result that the PCT even exceeds the one from the NA economy. Finally, for high-productivity agents altruistic-links matter little and so here the resemblance of the NA and IA economies is particularly pronounced.

4.6 PCT decomposition In order to provide further insights behind the size of the PCT at times other than at the announcement and the implementation dates, it is useful to decompose the PCT. In order to economize on space, I concentrate here on the short-term experiment. The results for the other experiments are (mostly) similar.

Aggregate consumption during the experiment differs from its stationary value because there are changes to agents' consumption policies and to the distribution of agents over the state-space. These two forces can be separated and the PCT can be decomposed as follows, see Section A.5 in the appendix for details on this decomposition,

$$\mathrm{PCT}_t = \frac{C_t - \bar{C}}{\Delta Y_t^d} \approx \underbrace{\frac{\Delta\text{policy}}{\Delta Y_t^d}}_{\text{PCT}-\Delta\text{policy}} + \underbrace{\frac{\Delta\text{density}}{\Delta Y_t^d}}_{\text{PCT}-\Delta\text{density}}, \tag{11}$$

where C_t is aggregate consumption at time t and $\bar{C}$ is stationary aggregate consumption. The term Δpolicy captures the effect on aggregate consumption from changes in agents' consumption policies under the stationary density, while Δdensity contains the consequences on aggregate consumption due to changes in the density maintaining the stationary consumption policy. The size of the PCT_t is approximately the sum of these two terms. Dividing these terms by ΔY^d ensures that the contributions are to the PCT.

Figure 5(a) shows decomposition (11) for the short-term experiment: The thick solid line is the actual PCT; the thin solid line is the PCT-Δpolicy component and the dashed line is the PCT-Δdensity component. Figure 5(b) depicts the Δpolicy component of unconstrained agents.

4.6.1 PCT-Δpolicy Between 2000 and 2001 the PCT is positive only because of the term PCT-Δpolicy (the thin line and the solid line coincide). This component is forward-looking, and contains the change in consumption of unconstrained agents in response to the announcement effect. In 2001, the PCT-Δpolicy component jumps up due to the change in consumption of constrained agents in response to the impact effect (again, the thin line and the solid line coincide), at least, for those who are unaffected by the crowding-out of private transfers. For agents who continue to be constrained, the PCT-Δpolicy term remains positive for the duration of the tax-cut regime (i.e. the difference in their disposable income times their mass in steady state), and so constrained agents contribute positively to the PCT. For unconstrained agents, however, this term becomes negative. Figure 5(b) zeros in on the consumption policies of unconstrained agents by showing their Δpolicy component. This term rapidly turns negative (the time at which this term turns negative happens later for the experiments of longer durations) and decreases throughout the tax-cut regime. Thus, unconstrained agents engage in additional savings in anticipation of higher taxes, though depending on the economy, they do so to differing degrees. PA-economy agents accumulate most, followed by agents in the NA economy, while IA economy-agents do so least.

Taking the contribution of constrained agents and unconstrained agents together, explains why the PCT-Δpolicy component (the thin line) is decreasing throughout the tax-cut regime. It also explains why this term decreases more rapidly in the PA and NA economies than in the IA economy.

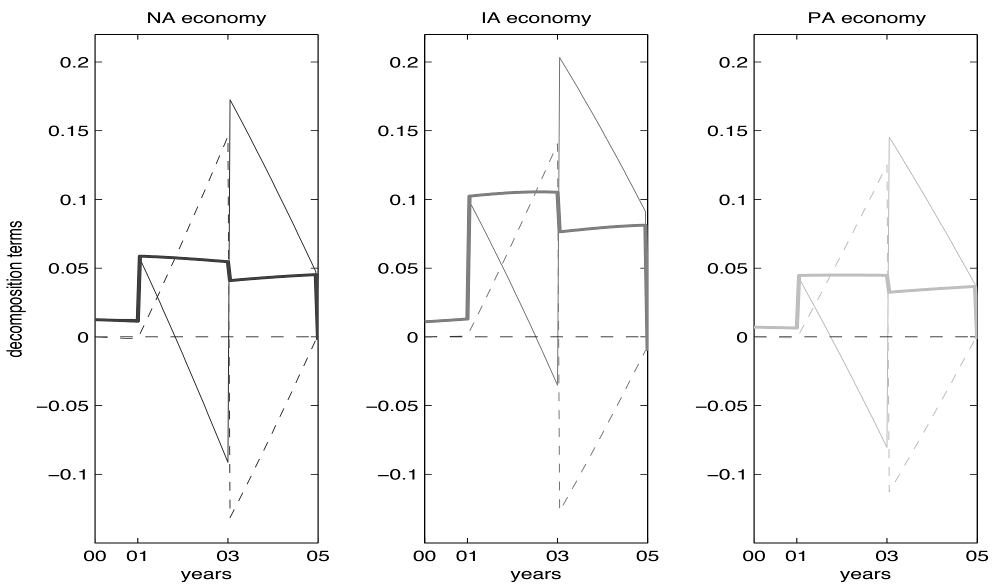
So far, we have analyzed why the PCT-Δpolicy component in decomposition (11) of the PCT is decreasing and eventually becomes negative. But, from Figure 5(a) we see that the PCT (the thick solid line) neither decreases nor becomes negative. The countervailing force is captured by the PCT-Δdensity component.

4.6.2 PCT-Δdensity The PCT-Δdensity term is depicted as the dashed line in Figure 5(a). This term is backward-looking and takes into account past decisions about consumption on the stock of wealth and changes only gradually. This component of the PCT increases and it does so for two closely related reasons.

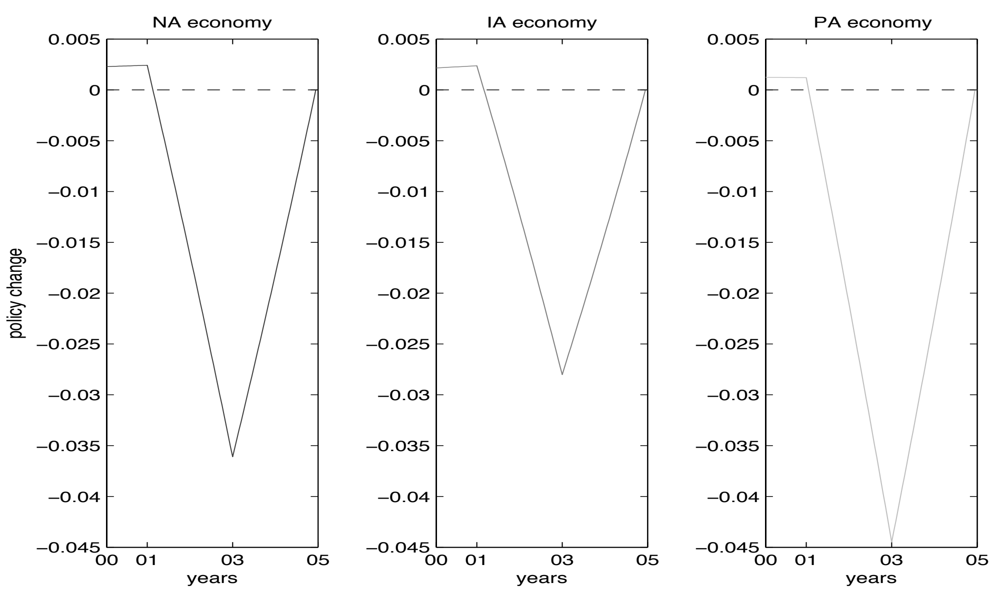
The fraction of constrained agents decreases bit by bit as some become unconstrained and so the mass of agents without wealth is smaller under the time-t density than under the stationary one. Similarly, unconstrained agents engage in additional savings so that an increasingly larger mass of agents sits in higher wealth states than under the stationary density. As agents move into higher wealth states, the stationary consumption policy, which is contained in the PCT-Δdensity term, prescribes higher consumption rates, so that PCT-Δdensity increases throughout the tax-cut regime. In other words, over time an increasing fraction of the PCT is explained simply by agents consuming more because they have more wealth.

Summing up the changes in policy and the changes in density, discussed so far, explains

Figure 5: Decomposition of short-term experiment



(a) PCT decomposition



(b) Δpolicy: unconstrained agents

(a) Shows decomposition of PCT, equation (11). Thick-solid line is PCT. Narrow-solid line is PCT-Δpolicy, and dashed line is PCT-Δdensity. (b) Shows Δpolicy component for unconstrained agents.

the decomposition of the PCT for the tax-cut regime.

4.6.3 Tax-hike regime The tax hike takes place in 2003. The change in aggregate disposable income is now negative, $\Delta Y^d < 0$, and in absolute terms larger than under the tax-cut regime; debt, including the accumulated interest, must be repaid. Figure 5(a) shows that the PCT jumps downward when entering the tax-cut regime and remains positive throughout. The decomposition clarifies why this is the case.

At the time of the tax hike, the PCT-Δpolicy component jumps up (the thin line). This is thanks to both constrained, and, maybe surprisingly, also unconstrained agents. First, for constrained agents, Δpolicy is positive prior to the tax-hike regime and becomes discontinuously negative once the tax hike takes place. Disposable income decreases which for constrained agents translates into a downward jump in consumption. However, when dividing Δpolicy for constrained agents by $\Delta Y^d < 0$ the term PCT-Δpolicy is positive. Second, for unconstrained agents the term Δpolicy is continuous and remains negative when they enter the tax-hike regime, as shown by Figure 5(b). But, because $\Delta Y^d < 0$ the term PCT-Δpolicy for unconstrained agents becomes positive as well.

Furthermore, Figure 5(a) shows that the PCT-Δdensity component (dashed line) becomes negative when entering the tax-hike regime. Prior to this regime, as agents accumulate wealth in anticipation of the tax hike, Δdensity is positive. This term remains positive since the density changes continuously. The sudden turn is simply because $\Delta Y^d < 0$.

The term PCT-Δpolicy outweighs PCT-Δdensity which explains why the PCT, the sum of those two components, continues to be positive throughout the tax-hike regime. As the economy heads towards the end of the tax-hike regime, unconstrained agents' time-t consumption gradually approaches stationary consumption from below, and therefore the PCT-Δpolicy term decreases throughout the tax-hike regime (the numerator becomes less negative). When the tax-hike regime ends, the tax rate is permanently reduced to its stationary value, and consumption policies revert to the stationary policies. As unconstrained agents save less in anticipation of lower taxes, aggregate wealth in the economy decreases and the density of households in states with lower wealth increases. This is reflected by the PCT-Δdensity component.

4.7 Welfare changes induced by tax policy I now turn to a brief welfare analysis by calculating consumption equivalent variations (CEV) for the deficit-financed tax cut experiments. Here, CEV measure the percentage change of annual consumption an unborn agent under the veil of ignorance would require in the stationary equilibrium to be indifferent between the prospect of being born as a young agent in the year 2000 into the stationary equilibrium or into the economy with a deficit-financed tax cut. Thus, positive CEV indicate that such an agent requires compensation of being born into the stationary equilibrium and so indicates that the economy with a change in the timing of taxes is welfare-improving from an ex-ante perspective.

Table 3 shows CEV for the economies. The reason that this type of government policy is welfare-improving is because it can be undone by saving the tax cut. This change in the timing of taxes merely enlarges the set of feasible consumption paths an agent can choose from but does not force agents to change their consumption behavior.[24]

Table 3: CEV (%)

duration	NA	IA	PA
short	0.0079	0.0039	0.0027
medium	0.0231	0.0097	0.0058
half-generation	0.1142	0.0525	0.0337
full-generation	0.2419	0.1445	0.1024

Consumption equivalent variations for agents under the veil of ignorance.

From an ex-ante perspective this type of financing policy is valued more the longer the tax-cut regime lasts. As the duration lengthens the tax cut becomes more and more an increase in permanent income. This is especially true for the NA and IA economies in which agents have no or little concern for future generations and least for the PA economy. The likelihood that borrowing constraints will bind during the experiment goes up and so agents in the PA economy also increasingly value this type of policy. Interestingly, now the IA economy does lie between the extreme cases of altruism. A priori, this is what we would have expected. But, with the benefit of the prior analysis, it now seems puzzling that changes to welfare do not exceed those of the NA economy. In fact, the economy's welfare implications, gravitate closer to the PA economy's.

[24]Nonetheless, there are losers from this policy: first, of course, unborn agents who do not benefit from tax cuts but have to pay higher taxes. But there are also losers among agents alive in the year 2000. An agent with currently low productivity experiences an increase in the present value of expected taxes. With a certain probability this agent will have higher labor earnings by the time the tax hike is implemented. Since there is a proportional tax rate the tax amount also increases. On the other hand, the force that works against this is that low productivity agents are also more likely to be borrowing-constrained.

Table 4: CEV (%) by wealth percentile

wealth percentile	NA	IA	PA
p25	0.3342	0.1657	0.1382
p50	0.2353	0.1788	0.0751
p75	0.1625	0.1179	0.0412
p95	0.1045	0.0591	0.0281

Full-generation experiment. Consumption equivalent variations for agents across the stationary wealth distribution, averaging over productivity levels weighted by the stationary density.

This is due to the crowding-out effect. A consequence of the tax cut is that transfer recipients receive lower transfers since they now have higher disposable income. Thus, the tax cut crowds out gifts which is undesirable from the point of view of the transfer recipient: gifts are free of any future obligation of repayment while a tax cut is tied to the future financial burden of repaying the accumulated debt with higher taxes. Crowding out leaves the consumption profile of these households unaltered during the tax-cut regime, and hence, they do not collect additional utility, whereas the future financial obligation lowers their consumption. This implies a welfare loss that is absent from the NA economy, and so along this dimension, the IA economy loses its resemblance to this economy.

Table 4 focuses on the full-generation experiment and shows CEV of young agents at different percentiles of the stationary wealth distribution. There are two noteworthy observations. Firstly, the CEV in the IA economy at the median (p50) of the stationary wealth distribution is slightly higher than the one at the 25th percentile, whereas it is decreasing in wealth when moving to higher percentiles. Also, for the two other economies it is monotonically decreasing when moving from the low to the high percentiles. Secondly, it is primarily the CEV at the 25th percentile of the wealth distribution which is responsible of moving this economy closer to the PA economy along the welfare dimension. Both of these observations are once again due to the crowding-out effect. It is precisely at the 25th percentile of the wealth distribution where almost all gifts flow. Since agents here are negatively affected by the swap of the tax cut for gifts the benefit of relaxing the borrowing constraint is dampened.

4.7 Discussion I now turn to a brief discussion. In the data, transfers from the young to the old are small. Given this small magnitude, it is interesting to ask whether the inclusion of these transfers matter quantitatively for the results. Section B.3 in the appendix shows the calibration when setting altruism of the young to zero and summarizes results on the PCT. Quantitatively, there are only very minor changes, both in the calibration and the results. On a theoretical note, though, it is worthwhile to point out that even if altruism of the young towards the old is absent, the fundamental nature of the IA economy does not change. Strategic considerations are still present since the young have to keep track of the old's wealth in order to predict how many transfers they can expect.

Also, when increasing the targeted values of the gift-to-wealth ratio within an empirical plausible range the results change quantitatively only little.

A shortcoming of the current setting is that labor supply is inelastic. Clearly, this is not the case in the real world and one may wonder in which direction the results would change were labor supply endogenous. Presumably, a cut in the labor-income tax would lead to an increase in labor supply in all of the economies and, thus, since agents behave non-Ricardian, to higher consumption. This effect would likely be more pronounced in the NA and PA economies. In the IA economy agents face a disincentive to work since additional income is "taxed" by the donor through a decrease in private transfers.

5 Conclusions

To conclude, I provide a quantitative analysis of how government transfers interact with private transfers, by studying deficit-financed tax cuts of various durations. I find that the aggregate consumption response is large and resembles the one generated by an economy without altruistic linkages. Welfare implications, however, move closer to an economy with perfect altruism. One take-away of my analysis is that a standard OLG framework is preferable to study behavior of aggregate variables while a dynastic model is more appropriate for a welfare analysis.

However, the mechanisms responsible for the similarities of the IA economy to the standard workhorse models in macroeconomics are absent from these economies. The IA economy predicts more constrained households but also crowding-out of transfers. Thus, if a policy maker wants to know how strong a consumption response to expect from a tax cut, the fraction of transfer recipients has to be taken into account in addition to the fraction of constrained households. Furthermore, crowding out should be expected to be more severe if a tax relief is directed at younger segments of the population since these are also more likely to be recipients of gifts. Such a government policy could prove to be merely expensive without achieving a stimulative effect on consumption. In terms of changes to welfare the resemblance to the NA economy dwindles because for recipients of gifts permanent income can decrease. Thus, especially poor (constrained) households, often younger people, may be worse off when they receive a deficit-financed tax cut.

In future research, it could be interesting to study the effects of changes in government expenditures, using various financing schemes, to see how altruism affects the government expenditure multiplier. One could also revisit an earlier literature on how imperfect altruism, in conjunction with interesting sources of heterogeneity, matters for the consequences of other redistributive policies as, for example, changes to the Social Security system studied by Laitner (1988) or Altig & Davis (1993). One mechanism highlighted by this paper should lead to different results on the intergenerational allocation than previous studies: gifts are different from intergenerational transfers forced by the government (taxes, pensions) in a no-commitment world. Gifts need not be given back, whereas it is probably reasonable to think that the government has more commitment for transfers than the family.

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A Appendix

This appendix provides derivations of the HJB and the Euler equation. It also provides additional background on the payoff functions and the decomposition of the PCT.

A.1 Payoff functions in integral form

The present values of life-time utility for young and old agents are

$$V^y = \frac{1}{(1-\alpha^o\alpha^y)}\mathbb{E}_0\Big[\int_0^\tau \underbrace{e^{-\rho t}[u(c_t^y)+\alpha^y u(c_t^o)]\mathrm{d}t}_{\text{young stage}} + \underbrace{e^{-\rho\tau}W(\tau, A_\tau, \epsilon_\tau)}_{\text{value of aging}}\Big], \tag{12}$$

$$V^o = \frac{1}{(1-\alpha^o\alpha^y)}\mathbb{E}_0\Big[\int_0^\tau \underbrace{e^{-\rho t}[u(c_t^o)+\alpha^o u(c_t^y)]\mathrm{d}t}_{\text{old stage}} + \underbrace{e^{-\rho\tau}\alpha^o W(\tau, A_\tau, \epsilon_\tau)}_{\text{continuation value}}\Big], \tag{13}$$

where W is given by equation (4) and $A_\tau = a_\tau^y + a_\tau^o$ is wealth of the young upon death of the old. The uncertain time of death of the old agent is denoted by τ and follows an exponential distribution with Poisson rate δ. The rate-of-time preference is denoted by ρ. The present value of life-time utility is decomposed into what accrues over the current life-cycle stage and the instantaneous value of a change in the life-cycle stage.

Payoff functions (12) and (13) may suggest that an altruistic agent only cares about the current consumption flow of the other. However, one may argue, an altruistic agent's lifetime utility should take into account lifetime utility of the other. To see that these payoff functions actually do so consider a recursive representation of lifetime utility. Denote old and young agents' lifetime utility by V^o and V^y, and define utility from own consumption by

$$\tilde{U}^o = \int_0^\tau e^{-\rho t} u(c_t^o)\mathrm{d}t \quad \text{and} \quad \tilde{U}^y = \int_0^\tau e^{-\rho t} u(c_t^y)\mathrm{d}t.$$

For an altruistic agent lifetime utility is given by utility from own consumption and lifetime utility of the other weighted by the degree of altruism. For the old agent we have

$$V^o = \mathbb{E}_0\big[\tilde{U}^o + \alpha^o V^y\big], \tag{14}$$

and for the young agent we have

$$V^y = \mathbb{E}_0\big[\tilde{U}^y + e^{-\rho\tau} W + \alpha^y V^o\big], \tag{15}$$

where $e^{-\rho\tau}W$ summarizes the instantaneous value of becoming old. When substituting (14) into (15) we obtain payoff function (12). When substituting (15) into (14) we obtain payoff function (13). The scaling factor $1 - \alpha^o\alpha^y$ is irrelevant for the decision problem and is left out throughout the paper.

A.2 Derivation of HJBs

I derive the HJB for the old agent given by equation (5) in two steps. First I include only mortality hazard and second I add earnings uncertainty. The derivation of the young agent's HJB (2) is entirely analogous.

A.2.1 Only mortality hazard

The uncertain time of death of the old agent is denoted by τ and follows an exponential distribution with Poisson rate δ. Thus, the probability of death at time t is given by $\pi(t) = \delta e^{-\delta t}$. Take as given a function $W(t, A_t)$ which gives us a terminal value of death. Since the time of death is the only source of uncertainty consumption and assets are a deterministic function of time. Define the following function

$$v^o(\tau) = \int_0^\tau e^{-\rho t}[u(c_t^o) + \alpha^o u(c_t^y)]\mathrm{d}t + \alpha^o e^{-\rho\tau} W(\tau, A_\tau).$$

The expected value of v^o is

$$\mathbb{E}_0\Big(v^o(\tau)\Big) = \int_0^\infty \pi(t)\Big(\int_0^t e^{-\rho s}[u(c_s^o) + \alpha^o u(c_s^y)]\mathrm{d}s + \alpha^o e^{-\rho t} W(t, A_t)\Big)\mathrm{d}t.$$

When using integration-by-parts this equation becomes

$$\mathbb{E}_0\Big(v^o(\tau)\Big) = \int_0^\infty e^{-\rho t}\Big(e^{-\delta t}[u(c_t^o) + \alpha^o u(c_t^y)] + \pi(t)\alpha^o W(t, A_t)\Big)\mathrm{d}t,$$

Substituting $\pi(t) = \delta e^{-\delta t}$ we get

$$\mathbb{E}_0\Big(v^o(\tau)\Big) \equiv V^o = \int_0^\infty e^{-(\rho+\delta)t}\Big[u(c_t^o) + \alpha^o u(c_t^y) + \delta\alpha^o W(t, A_t)\Big]\mathrm{d}t. \tag{16}$$

Equation (16) is equivalent to equation (13) if only mortality hazard is present. The rate-of-time preference is now $\rho + \delta$ and the continuation value is multiplied by the death hazard. Intuitively, the probability of death conditional on surviving over a short time interval Δt is approximately $\delta\Delta t$ and $\alpha^o W$ contains the value conditional on this event.

We now use the principle of optimality on equation (16) to obtain the corresponding HJB. For ease of writing let $U(c^o, c^y) = u(c^o) + \alpha^o u(c^y)$. At time 0 the state is $(0, a_0^y, a_0^o)$ and we can write (16) as

$$\begin{aligned} V^o(0, a_0^y, a_0^o) = &\int_0^{\Delta t} e^{-(\rho+\delta)s}\Big[U(c_s^o, c_s^y) + \delta\alpha^o W(s, A_s)\Big]\mathrm{d}s + \\ &+ e^{-(\rho+\delta)\Delta t}\int_{\Delta t}^\infty e^{-(\rho+\delta)(s-\Delta t)}\Big[U(c_s^o, c_s^y) + \delta\alpha^o W(s, A_s)\Big]\mathrm{d}s. \end{aligned}$$

Approximating the first integral over the time interval $[0, \Delta t]$ and noting that the second integral is equation (16) at time Δt we get a recursive representation of the old agent's payoff function

$$V^o(0, a_0^y, a_0^o) \approx U(c_0^o, c_0^y)\Delta t + e^{-(\rho+\delta)\Delta t}\delta\Delta t\alpha^o W(\Delta t, A_{\Delta t}) + e^{-(\rho+\delta)\Delta t} V^o(\Delta t, a_{\Delta t}^y, a_{\Delta t}^o).$$

Next, approximate $e^{-(\rho+\delta)\Delta t}$ by $1 - \rho\Delta t - \delta\Delta t$, drop terms of order $(\Delta t)^2$, subtract $V^o(0, a_0^y, a_0^o)$

and divide by Δt to get

$$0 \approx -\rho V^o(\Delta t, a^y_{\Delta t}, a^o_{\Delta t}) + U(c^o_0, c^y_0) + \frac{V^o(\Delta t, a^y_{\Delta t}, a^o_{\Delta t}) - V^o(0, a^y_0, a^o_0)}{\Delta t} \tag{17}$$
$$+\delta[\alpha^o W(\Delta t, A_{\Delta t}) - V^o(\Delta t, a^y_{\Delta t}, a^o_{\Delta t})].$$

Now take a second-order Taylor approximation of $V^o(\Delta t, a^y_{\Delta t}, a^o_{\Delta t})$ in Δt around $(0, a^y_0, a^o_0)$ and divide by Δt,

$$\begin{aligned}\frac{V^o(\Delta t, a^y_{\Delta t}, a^o_{\Delta t}) - V^o(0, a^y_0, a^o_0)}{\Delta t} \approx\ & V^o_t(0, a^y_0, a^o_0) + \\ & + V^o_{a^y}(0, a^y_0, a^o_0)\dot{a}^y_0 + V^o_{a^o}(0, a^y_0, a^o_0)\dot{a}^o_0 + V^o_{tt}(0, a^y_0, a^o_0)\Delta t + \\ & + V^o_{a^y a^y}(0, a^y_0, a^o_0)\dot{a}^y_0 \Delta t + V^o_{a^o a^o}(0, a^y_0, a^o_0)\dot{a}^o_0 \Delta t + \\ & + 2V^o_{a^o a^y}(0, a^y_0, a^o_0)\dot{a}^o_0 \dot{a}^y_0 \Delta t\end{aligned}$$

Note, when taking the limit as $\Delta t \to 0$ only the first-order terms remain and thus it is enough to continue with the following first-order Taylor approximation

$$\frac{V^o(\Delta t, a^y_{\Delta t}, a^o_{\Delta t}) - V^o(0, a^y_0, a^o_0)}{\Delta t} \approx V^o_t(0, a^y_0, a^o_0) + V^o_{a^y}(0, a^y_0, a^o_0)\dot{a}^y_0 + V^o_{a^o}(0, a^y_0, a^o_0)\dot{a}^o_0 \tag{18}$$

Substitute (18) into (17) to get

$$\begin{aligned}\rho V^o(\Delta t, a^y_{\Delta t}, a^o_{\Delta t}) \approx U(c^o_0, c^y_0) + V^o_t(0, a^y_0, a^o_0) + V^o_{a^y}(0, a^y_0, a^o_0)\dot{a}^y_0 + V^o_{a^o}(0, a^y_0, a^o_0)\dot{a}^o_0 + \\ +\delta[\alpha^o W(\Delta t, A_{\Delta t}) - V^o(\Delta t, a^y_{\Delta t}, a^o_{\Delta t})].\end{aligned}$$

Finally, take the limit as $\Delta t \to 0$, and add the $\max$-operator to get the HJB of the old agent if there is only the risk of death

$$\rho V^o = V^o_t + \max_{c^o \geq 0, g^o \geq 0} \Big\{ U(c^o, c^y) + \dot{a}^o V^o_{a^o} + \dot{a}^y V^o_{a^y} + \delta[\alpha^o W - V^o] \Big\}. \tag{19}$$

A.2.2 Adding income uncertainty

Since labor productivity is also a Poisson process it can be included in the exact same manner as just shown for mortality hazard.

For clarity assume that only the old agent is subject to the labor-productivity shock. Also, suppose that labor productivity can either be high, ϵ_h, or low, ϵ_l, with Poisson-switching rate

σ. The probability of switching from one labor productivity to another over a short time interval Δt is approximately $\sigma\Delta t$. Assume that current labor productivity is high. The old agent's recursive Δt-formulation, after dropping terms of order $(\Delta t)^2$, is then given by

$$\begin{aligned} V^o(0, a_0^y, a_0^o, \epsilon_h) =& U(c_0^o, c_0^y)\Delta t + \delta\Delta t\alpha^o W(\Delta t, A_{\Delta t}) + \sigma\Delta t V^o(\Delta t, a_{\Delta t}^y, a_{\Delta t}^o, \epsilon_l) + \\ &+ (1 - \rho\Delta t - \delta\Delta t - \sigma\Delta t)V^o(\Delta t, a_{\Delta t}^y, a_{\Delta t}^o, \epsilon_h). \end{aligned}$$

Re-organizing this equation and dividing by Δt yields

$$\begin{aligned} \rho V^o(\Delta t, a_{\Delta t}^y, a_{\Delta t}^o, \epsilon_h) =& U(c_0^o, c_0^y) + \frac{V^o(\Delta t, a_{\Delta t}^y, a_{\Delta t}^o, \epsilon_h) - V^o(0, a_0^y, a_0^o, \epsilon_h)}{\Delta t} + \\ &+ \delta[\alpha^o W(\Delta t, A_{\Delta t}) - V^o(\Delta t, a_{\Delta t}^y, a_{\Delta t}^o, \epsilon_h)] + \\ &+ \sigma[V^o[(\Delta t, a_{\Delta t}^y, a_{\Delta t}^o, \epsilon_l) - V^o(\Delta t, a_{\Delta t}^y, a_{\Delta t}^o, \epsilon_h)]. \end{aligned}$$

Now substitute Taylor approximation (18) and take the limit as $\Delta t \to 0$ to obtain the HJB

$$\rho V^o = V_t^o + \max_{c^o \geq 0, g^o \geq 0} \Big\{ U(c^o, c^y) + \dot{a}^o V_{a^o}^o + \dot{a}^y V_{a^y}^o + \sigma\Big[V^o(\cdot, \epsilon_l) - V^o(\cdot, \epsilon_h)\Big] + \delta\Big[\alpha^o W - V^o\Big]\Big\}.$$

It is now straightforward to see how we can obtain equation (5) as well as the young agent's HJB (2).

A.3 Derivation of Euler equations

I will now derive Euler equation (7) for the old agent. Define the *infinitesimal generator* $\mathcal{A}$ for a function $f(t, x)$ that is differentiable in the continuous state variables (t, a^y, a^o) as

$$\mathcal{A}f(t, x_t) \equiv \lim_{\Delta t \to 0} \frac{1}{\Delta t} E_t[f(t + \Delta t, x_{t+\Delta t}) - f(t, x_t)].$$

Thus, this generator is the expected change of the function f over an instant of time. I will now show that the operator $\mathcal{A}$ in my model is given by

$$f_t + f_{a^y}\dot{a}_t^y + f_{a^o}\dot{a}_t^o + \sum_j e(i^y, j)f(\cdot, \epsilon_j) + \sum_j e(i^o, j)f(\cdot, \epsilon_j),$$

where subscripts refer to partial derivatives, f is either a value function or a marginal value function, and e is an element of the hazard matrix (3).

First, let us consider the case when f is a value function. For clarity assume again that only the old agent is subject to the labor-productivity shock, that it can either be high or low, and that the Poisson-switching rate is σ. The expected change of the old agent's value function with respect to labor productivity over a short time interval Δt is given by

$$\begin{aligned}
&\mathbb{E}_t\Big[V^o(t+\Delta t, x_{t+\Delta t}) - V^o(t, x_t)\Big] = \\
&\quad = \underbrace{\sigma\Delta t}_{\approx \text{pr of switch}} \; V^o(t+\Delta t, \tilde{x}_{t+\Delta t}) + \underbrace{(1-\sigma\Delta t)}_{\approx \text{pr of staying}} V^o(t+\Delta t, x_{t+\Delta t}) - V^o(t, x_t) = \\
&\quad = \sigma\Delta t\Big[V^o(t+\Delta t, \tilde{x}_{t+\Delta t}) - V^o(t+\Delta t, x_{t+\Delta t})\Big] + V^o(t+\Delta t, x_{t+\Delta t}) - V^o(t, x_t),
\end{aligned}$$

where I use short-hand notation $\tilde{x}$ to denote the entire state when productivity of the old agent has changed. Divide this equation by Δt, substitute Taylor approximation (18), and take the limit as Δt goes to zero

$$\begin{aligned}
\lim_{\Delta t \to 0} \frac{1}{\Delta t} E_t[V^o(t+\Delta t, x_{t+\Delta t}) - V^o(t, x_t)] =& V^o_t(t, x_t) + \dot{a}^o V^o_{a^o}(t, x_t) + \dot{a}^y V^o_{a^y}(t, x_t) + \\
&+ \sigma\Big[V^o(t, \tilde{x}_t) - V^o(t, x_t)\Big].
\end{aligned}$$

Note that $\tilde{x}_t$ in the last term equals x_t for all components of the vector, because asset paths are continuous, except for the income state. The HJB of the old agent is

$$\underbrace{V^o_t + \dot{a}^o V^o_{a^o} + \dot{a}^y V^o_{a^y} + \sigma\Big[V^o(\cdot, \tilde{\epsilon}) - V^o(\cdot, \epsilon)\Big]}_{=\mathcal{A}V^o} = \rho V^o - u(c^o) - \alpha^o u(c^y) - \delta\Big(\alpha^o W - V^o\Big),$$

where I have substituted optimal policies.

In order to find the Euler equation differentiate this HJB with respect to wealth of the old

$$\begin{aligned}
&\underbrace{V^o_{ta^o} + \dot{a}^o V^o_{a^o a^o} + \dot{a}^y V^o_{a^y a^o} + \sigma\Big[V^o_{a^o}(\cdot, \tilde{\epsilon}) - V^o_{a^o}(\cdot, \epsilon)\Big]}_{=\mathcal{A}V^o_{a^o}} + (r - c^o_{a^o})V^o_{a^o} + (r - c^y_{a^o})V^o_{a^y} = \\
&\qquad = \rho V^o_{a^o} - u_c(c^o)c^o_{a^o} - \alpha^o u_c(c^y)c^y_{a^o} - \delta\Big(\alpha^o W_A - V^o_{a^o}\Big).
\end{aligned}$$

Note, when differentiating the laws of motion with respect to a^o, partial derivatives $g^o_{a^o}$ and $g^y_{a^o}$ arise. I neglect these in the presentation of the Euler equations since gift functions are zero within the state space. Now, group the common terms in the following way

$$\mathcal{A}V^o_{a^o} = (V^o_{a^o} - u_c(c^o))c^o_{a^o} + (\rho - r)V^o_{a^o} + [V^o_{a^y} - \alpha^o u_c(c^y)]c^y_{a^o} - \delta\Big(\alpha^o W_A - V^o_{a^o}\Big),$$

and substitute first-order condition $u_c(c^o) = V^o_{a^o}$ to get the Euler equation of the old agent

$$\mathcal{A}u_c(c^o) = (\rho - r)u_c(c^o) + [V^o_{a^y} - \alpha^o u_c(c^y)]c^y_{a^o} - \delta\Big(\alpha^o W_A - V^o_{a^o}\Big).$$

The Euler equation for the young agent can be obtained following the exact same steps.

A.4 PCT

The propensity to consume out of income taxes is defined as

$$\text{PCT}_t = \frac{C_t - \bar{C}}{\Delta Y^d_t} = \frac{\int_X \Big[(c^y(t,x) + c^o(t,x))\lambda(t,x) - (\bar{c}^y(x) + \bar{c}^o(x))\bar{\lambda}(x)\Big]\mathrm{d}x}{(\phi_t - \bar{\phi})Y},$$

where $\int_X \mathrm{d}x$ means integration over the state space X, $\bar{c}$'s are stationary consumption functions, $\bar{\lambda}$ is the stationary measure, and Y is aggregate output. Functions with a time argument are those of the experiment and ϕ_t is the relevant tax rate.

A.5 PCT decomposition

I now show how to decompose aggregate consumption, and hence the PCT, during an experiment. The idea is to separate the consumption response into the contribution due to changes in agents' optimal consumption policies and the contribution which arises since the measure of agents changes.

Define the following objects

$$\tilde{c}(t,x) = c(t,x) - \bar{c}(x), \quad \text{and} \quad \tilde{\lambda}(t,x) = \lambda(t,x) - \bar{\lambda}(x),$$

and compute the integrals

$$\bar{C} = \int_X \bar{c}(x)\bar{\lambda}(x)\mathrm{d}x, \quad C_t = \int_X c(t,x)\lambda(t,x)\mathrm{d}x, \quad \tilde{C}_t = \int_X \tilde{c}(t,x)\bar{\lambda}(x)\mathrm{d}x,$$
$$\tilde{\Lambda}_t = \int_X \tilde{\lambda}(t,x)\bar{c}(x)\mathrm{d}x, \quad \text{and} \quad \tilde{C\Lambda}_t = \int_X \tilde{c}(t,x)\tilde{\lambda}(t,x)\mathrm{d}x \quad \forall t.$$

Note that,

$$\tilde{C}_t + \tilde{\Lambda}_t + \tilde{C\Lambda}_t = \int_X \Big[\tilde{c}(t,x)\bar{\lambda}(x) + \tilde{\lambda}(t,x)\bar{c}(x) + \tilde{c}(t,x)\tilde{\lambda}(t,x)\Big]\mathrm{d}x = C_t - \bar{C},$$

and so aggregate consumption at time t can be decomposed as follows

$$C_t = \bar{C} + \tilde{C}_t + \tilde{\Lambda}_t + \tilde{C\Lambda}_t.$$

In the computations I find that the last term is negligible, which is to be expected since it is an order of magnitude lower than the other terms, and we can write

$$C_t - \bar{C} \approx \tilde{C}_t + \tilde{\Lambda}_t = \Delta\text{policy} + \Delta\text{density}.$$

The difference between aggregate consumption during the experiment, C_t, and stationary aggregate consumption, $\bar{C}$, is given by the effects that stem from changes in policies, evaluated under the stationary density, and those from changes in the density, evaluated under the stationary consumption policy. When dividing the decomposition by the tax change we obtain the PCT and its decomposition

$$\text{PCT}_t = \frac{C_t - \bar{C}}{\Delta Y_t^d} \approx \frac{\tilde{C}_t}{\Delta Y_t^d} + \frac{\tilde{\Lambda}_t}{\Delta Y_t^d},$$

as claimed by equation (11).

B Appendix

This appendix defines the numerical algorithm. It is closely related to value function iteration in discrete time. The solution method consists of iterating backward through time on the system of HJBs given by equations (2) and (5). The law of motion for the state is approximated by a Markov chain that is locally restricted to adjacent grid points. In order to obtain the density, the mass of agents is mapped forward governed by the agents' laws of motion induced by the policies and the exogenous shock processes.

B.1 Numerical algorithm

1. Discretize wealth into a uniformly-spaced grid $[0, \Delta a, 2\Delta a, \ldots, \bar{a}]$ with mesh size Δa, N grid points, and maximal level of wealth $\bar{a} = (N-1)\cdot\Delta a$.

2. Discretize time into small intervals of size Δt. This time increment has to be chosen small enough in order for the CFL condition to hold.[25] To initialize the backward-iteration procedure construct a reasonable consumption allocation over an initialization period $[T, T+s]$, where T is some distant point in the future and s is a short time interval. Obtain value functions at time $t = T$.

   Stationary equilibrium.

3. Using the value functions at time $t = T$, go backward in time by Δt, where Δt has to satisfy the CFL condition.

   (a) On grid points where both agents are unconstrained obtain consumption from the first-order conditions $u_c(c^y) = V^y_{a^y}$ and $u_c(c^o) = V^y_{a^o}$ and set gifts to zero $g^y = 0 = g^o$. Partial derivatives $V^o_{a^o}$ and $V^o_{a^y}$ are approximated by using a finite-difference approximation. When the old (young) agent is saving, $\dot{a}^o > 0$ ($\dot{a}^y > 0$), the forward difference is used for the approximation, and otherwise the backward difference is employed (see *upwind differencing* in Chapter 8 in Marimon & Scott, 1999). On grid points where the young or the old agent is broke, the determination of consumption and gifts follows the procedure outlined in Barczyk & Kredler (2014*a*), Section 3.3 and Appendix A.2.

   (b) The function $V^o(\cdot, a^y + a^o)$ needs to be evaluated for levels of wealth that lie far outside of the wealth grid because of bequests. An excellent way of doing this is to exploit homogeneity: for families with large levels of wealth the importance of income becomes negligible. Thus, policy functions are well approximated by assuming them to be linear in total wealth $A = a^y + a^o$. In a homogeneous environment the old agent's value function takes on the form

$$V^o(t, a^y, a^o, \epsilon^y, \epsilon^o) = \tilde{v}(t, P)A^{1-\gamma}, \quad \text{where} \quad P = \frac{a^o}{A}.$$

[25] This condition is well-known from the theory on finite-difference solution methods for partial differential equations. It ensures that the interpolation weights defined in equation (21) can be interpreted in a stochastic sense, i.e. that they sum up to one; see Chapter 8 in Marimon & Scott, 1999 and Section S.1.3 of the online appendix in Barczyk & Kredler, 2014*a*.

$P \in [0,1]$ is the fraction of total wealth owned by the old agent. The function $\tilde{v}$ can then be calculated from

$$\tilde{v}(t,P) = V^o(t,a^y,a^o,\epsilon^y,\epsilon^o)A^{\gamma-1},$$

where the outermost grid points – the grid points where either the old, the young, or both hold the maximal level of wealth $\bar{a}$ – are used.

4. Updating step. With the value function at time $t+\Delta t$ linearly interpolate values off the grid as follows

$$\begin{aligned} V^o(t+\Delta t, a^y_{t+\Delta t}, a^o_{t+\Delta t}) \approx V^o(t+\Delta t, a^y, a^o)+ & \\ +\omega^o_+[V^o(t+\Delta t, a^y, a^o+\Delta a) - V^o(t+\Delta t, a^y, a^o)]+ & \\ +\omega^o_-[V^o(t+\Delta t, a^y, a^o-\Delta a) - V^o(t+\Delta t, a^y, a^o)]+ & \\ +\omega^y_+[V^o(t+\Delta t, a^y+\Delta a, a^o) - V^o(t+\Delta t, a^y, a^o)]+ & \\ +\omega^y_-[V^o(t+\Delta t, a^y-\Delta a, a^o) - V^o(t+\Delta t, a^y, a^o)], & \end{aligned} \tag{20}$$

where the interpolation weights, ω, are given by

$$\begin{aligned} \omega^o_+ &= \max\left\{\frac{\dot{a}^o\Delta t}{\Delta a},0\right\}, \quad \omega^o_- = \max\left\{-\frac{\dot{a}^o\Delta t}{\Delta a},0\right\} \\ \omega^y_+ &= \max\left\{\frac{\dot{a}^y\Delta t}{\Delta a},0\right\}, \quad \omega^y_- = \max\left\{-\frac{\dot{a}^y\Delta t}{\Delta a},0\right\}. \end{aligned} \tag{21}$$

Define the positive and negative parts of the savings policies as follows

$$\begin{aligned} \dot{a}^o_+ &= \max\left\{\dot{a}^o,0\right\}, \quad \dot{a}^o_- = \max\left\{-\dot{a}^o,0\right\} \\ \dot{a}^y_+ &= \max\left\{\dot{a}^y,0\right\}, \quad \dot{a}^y_- = \max\left\{-\dot{a}^y,0\right\}. \end{aligned}$$

Substitute equation (20), including interpolation weights (21), into equation (17). Divide by Δt and drop terms of order $(\Delta t)^2$ to obtain

$$\begin{aligned} -\Big[\frac{V^o(t+\Delta t, a^y, a^o) - V^o(t, a^y, a^o)}{\Delta t}\Big] \approx & -\rho V^o(t+\Delta t, a^y, a^o) + U(c^o, c^y) + \\ & + \delta\Big[\alpha^o V^o(t+\Delta t, 0, a^y + a^0) - V^o(t+\Delta t, a^y, a^o)\Big] + \\ & + \Big[\frac{V^o(t+\Delta t, a^y, a^o + \Delta a) - V^o(t+\Delta t, a^y, a^o)}{\Delta a}\Big]\dot{a}^o_+ + \\ & + \Big[\frac{V^o(t+\Delta t, a^y, a^o - \Delta a) - V^o(t+\Delta t, a^y, a^o)}{\Delta a}\Big]\dot{a}^o_- + \\ & + \Big[\frac{V^o(t+\Delta t, a^y + \Delta a, a^o) - V^o(t+\Delta t, a^y, a^o)}{\Delta a}\Big]\dot{a}^y_+ + \\ & + \Big[\frac{V^o(t+\Delta t, a^y - \Delta a, a^o) - V^o(t+\Delta t, a^y, a^o)}{\Delta a}\Big]\dot{a}^y_- \end{aligned} \tag{22}$$

Note, the limit of equation (22) as $\Delta t \to 0$ yields the old agent's HJB (19). The key insight here is that the HJB provides us with information about the change in the value function along the time dimension: the left-hand side of equation (22) is an approximation to partial derivative V_t. Thus, using a guess for the value function we can evaluate the right-hand side of equation (22) and then use the result to update the value function a Δt-period backwards in time: $V^o(t, a^y_t, a^o_t) \approx V^o(t+\Delta t, a^y, a^o) - V_t(t, a^y_t, a^o_t)\Delta t$.

5. Continue with steps 3 and 4 until value functions converge. Check whether the Lagrange multipliers for transfers for the young and the old are positive throughout the state space. If they are, the choice of setting gifts within the state space is justified and we have found optimal policies of the stationary equilibrium.

6. At $t = 0$ initialize the forward-iteration procedure with a density. Use the stationary policies obtained in step 5 to construct Markov transition probabilities to approximate the laws of motion on a trinomial lattice. These probabilities are given by the interpolation weights (21). Map the density forward using this Markov chain, the hazard matrix (3), and morality hazard δ; for an introduction to this forward equation and its connection to the *Kolomogorov forward equations* see Section S.1.8 in the Online Appendix of Barczyk & Kredler (2014*a*). The forward iteration stops when the density converges.

The methods which are used to solve for the stationary equilibrium can also be used to compute the experiments.

Policy experiments.

(a) Obtain agents' policies for the duration of the policy experiment. Return to step 3. Initialize the backward-iteration procedure at time $t = S_1 + S_2$ using the stationary value functions obtained previously from step 5. (Note, at the end of an experiment policies and value functions are given by the stationary ones because I assume a small open economy, inelastic labor supply, and the absence of aggregate uncertainty.) Under the new tax sequence iterate over steps 3 and 4. Check whether Lagrange multipliers for transfers are positive throughout the state space. Stop at time $t = S_0$.

(b) Obtain measures of agents over the state space for the duration of the policy experiment. Return to step 6. To initialize the forward-iteration procedure use the stationary density; to construct the Markov transition probabilities use policies obtained in (a). The forward iteration stops at time $t = S_1 + S_2$.

B.2 Altruism Parameter

I provide an economic interpretation of the altruism parameter and show that this parameter depends on the degree of risk-aversion.

Consider per-period utility function of old agent o, which is additively separable in own consumption and consumption of young agent y,

$$U^o(c^o, c^y) = u(c^o) + \alpha u(c^y), \quad \alpha \in [0, 1]$$

where $u(\cdot)$ is a CRRA utility. When the young agent is constrained the following first-order condition has to hold

$$u_c(c^o) \geq \alpha u_c(c^y) \quad \Rightarrow \quad \alpha \leq \left(\frac{c^o}{c^y}\right)^{-\gamma}.$$

When the margin is equalized we have that $c^y/c^o = \alpha^{1/\gamma}$ and so we can see that the degree of altruism can be interpreted as how much consumption inequality an altruist tolerates before providing gifts. Crucially, this tolerance depends also on the coefficient of relative risk-aversion unless $\alpha = 1$.

B.3 Discussion

Table 5 contrasts the calibration result of the benchmark IA economy with that obtained when setting the young gift-to-wealth ratio to zero. Young agents are not altruistic and so this economy is a one-sided altruism economy. The calibrated value of the old's degree of altruism remains unaffected. The rate-of-time preference increases slightly, reflecting the fact that old agents accumulate higher precautionary savings, since they cannot rely on the young to obtain transfers under certain circumstances. Thus, impatience has to increase as otherwise the calibration would overshoot the wealth-to-GDP ratio. Fewer old agents are constrained while the fraction of constrained young agents slightly increases. Overall, the fraction of constrained agents decreases. The fraction of young gift-recipients remains unchanged and by construction there are no old gift-recipients. The bequest-to-wealth ratio increases somewhat because the old save more.

Table 6 summarizes the quantitative results with respect to the PCT of the one-sided altruism economy and puts these results into contrast with the benchmark IA economy. For completeness, I also include the NA and PA economies. Part (a) of the table shows the PCT at the time of the impact of the tax cut. The quantitative differences between the IA and one-sided altruism economies are small. PCTs upon impact are slightly smaller for durations, other than the full-generation experiment, primarily because the fraction of constrained agents is lower. For the full-generation experiment, over-consumption appears to become stronger and outweighs the effect of more constrained agents. Part (b) of the table shows time averages of the PCT for the duration of the tax-cut regime. All the averages are lower in the one-sided altruism economy than in the benchmark economy. Again, quantitatively the differences are small.

To summarize, the one-sided altruism economy provides too few constrained households and not enough wealth-poor households. By construction, it cannot explain transfers from the young to the old. The consumption responds and welfare implications (not shown here) move slightly closer to the NA economy.

Table 5: One-sided altruism

	data	IA	one-sided
calibration target			
wealth/GDP	3.0	3.0	3.0
old gift/wealth	0.32%	0.32%	0.32%
young gift/wealth	0.03%	0.03%	0
parameter			
ρ		3.42%	3.52%
α^y		0.110	0
α^o		0.295	0.295
not targeted			
constrained households	19.0%	19.4%	17.9%
wealth-poorest 40%	3.1%	3.0%	3.4%
young gift-recipients	16.1%	15.3%	15.3%
old gift-recipients	5.4%	2.6%	0
bequest/wealth	1.06%	2.5%	2.6%

IA calibration as shown in Table 1 in contrast to calibration when assuming only old is altruistic.

Table 6: PCT summary

	IA	one-sided	NA	PA
(a) on impact				
short	10.2	9.5	5.9	4.5
medium	12.3	11.5	8.5	5.8
half-generation	25.6	25.1	25.1	15.9
full-generation	47.0	47.3	49.4	34.7
(b) on average				
short	10.4	9.7	5.7	4.5
medium	12.6	11.9	7.7	5.5
half-generation	24.3	23.5	19.9	12.2
full-generation	39.2	38.4	34.9	21.9

PCT summary in cents. Part (a) shows PCT at time of impact of policy in year 2001. Part (b) shows time-averaged PCT over the duration of the tax-cut regime starting from 2001.